

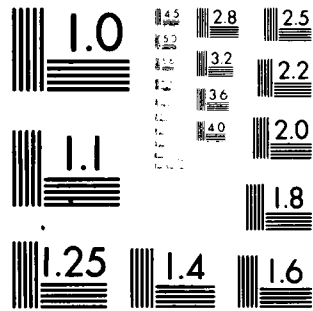
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AUG 81 J O MATSON, J A WHITE N00014-80-K-0709

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STORAGE SYSTEM OPTIMIZATION

by

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and

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PDRC 81-09

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## Storage System Optimization

### Abstract

This report focuses on optimization procedures for the design and evaluation of storage system alternatives, including block stacking, single-deep and double-deep pallet rack, and deep lane storage. The development and application of analytical models are demonstrated for the design of storage systems based on floor space utilization and handling time criteria.

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## Introduction

This study focuses on the design and selection of warehouse storage and handling systems.

Today, more than 278,000 businesses in the United States, which engage in wholesale trade, operate and maintain warehouses for production smoothing, customer service, or other reasons. These warehouses provide over three billion square feet of floor space for inventories valued in excess of \$75 billion [U.S. Department of Commerce, 1981].

The appropriate selection of a storage and handling system can play a major role in controlling warehousing costs. Thus, the increased emphasis on cost control in the 1980's makes it essential to develop valid procedures for evaluating storage system alternatives and to select those design alternatives which minimize costs and meet service level objectives.

In designing a storage and handling system, warehouse planners may consider numerous storage methods and equipment alternatives including block stacking, selective pallet racks, flow racks, automated storage and retrieval systems, shelving, deep lane storage systems, carousels, and many others. Selection of the appropriate storage method and equipment depends on a comparison of the costs and operational characteristics for the optimum designs of the various alternatives under consideration. However, procedures for determining optimum designs are not well developed for some storage methods. In particular, designs for deep lane storage and block stacking methods are often based on rules of thumb or the experience of the warehouse manager. Therefore, this study will concentrate on design

procedures for the following four storage alternatives:

- block stacking,
- single-deep selective pallet rack,
- double-deep selective pallet rack, and
- deep lane storage systems.

This report is divided into three major areas. The first focuses on definition of the block stacking design problem and provides a review of the related literature. The second area demonstrates the development of analytical models based on the criteria of space utilization and handling requirements. The last section examines computational results obtained from application of the storage models.

### Problem Description

#### Assumptions

The design and selection of a storage system is dependent on warehouse operating policies. Many storage situations can be described by the following set of characteristics.

1. The warehouse operation is based on a policy of randomized storage.
2. First-in-first-out (FIFO) lot rotation is required for all products.
3. Full pallet load storages and retrievals are required. Each pallet load represents one stock-keeping unit (SKU).
4. As a product lot is depleted, residual pallet loads are not relocated to obtain possible space savings.
5. The lot size for each product is known.

These characteristics represent the basic assumptions for this study.

#### Block Stacking

Block stacking involves the storage of unit loads in stacks (columns

one or more units high) within storage lanes (one or more stacks deep). This storage method is often used for the storage of appliances, food and beverages, household products, etc. For storage of full product lots, block stacking provides a high utilization of space at low cost. However, during the storage and retrieval cycle of a product lot, vacancies can occur in the storage lane. To achieve FIFO lot rotation, the vacant storage positions cannot be used for storage of other products or lots until all pallet loads have been withdrawn from the lane. The space losses resulting from unusable storage positions are referred to as "honeycomb loss"; block stacking suffers from both vertical and horizontal honeycombing. Figure 1 illustrates the space losses resulting from honeycombing.

The design of the block stacking storage system is characterized by

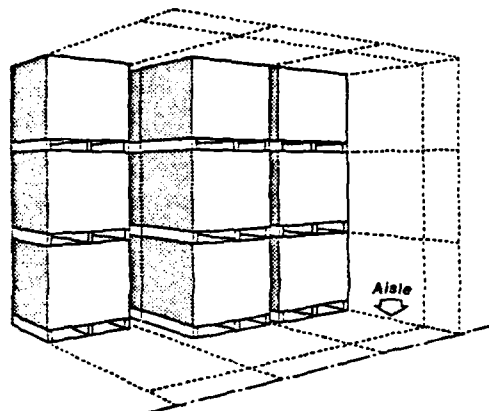
- the height of the stack (an integer number of pallet loads),
- the depth of the storage lane (an integer number of stacks), and
- the number of storage lanes required for a given product lot.

The key decision variable in the block stacking design problem is the storage lane depth. For a single product, factors which may influence the optimum lane depth include lot size, load dimensions, required aisle widths and clearances, stacking height, and the storage and retrieval distribution. Figure 2 illustrates feasible lane depths for a product with a lot size of fifteen and a stacking height of three pallet loads.

For multiple products, there are other decision variables. The design analysis must include a determination of the optimum number of unique lane depths, the values of those depths, the assignment of products to depths, and aggregate space requirements.

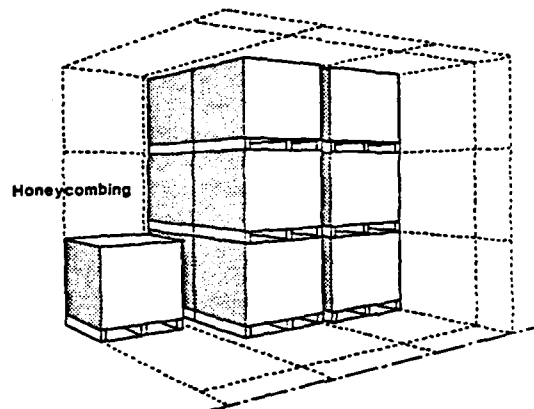
Generally deeper lanes imply that more of the total space is committed to storage versus aisles. Deeper lanes may also result in greater space

START...



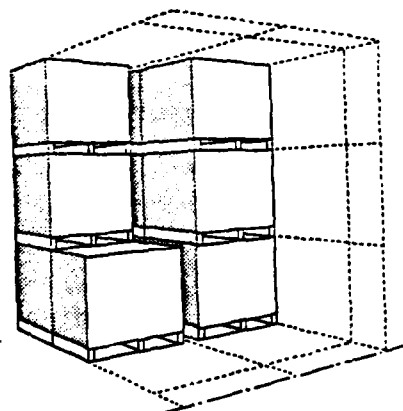
ON HAND—15 PALLET LOADS

LATER...



ON HAND—13 PALLET LOADS

STILL LATER...



ON HAND—10 PALLET LOADS

Figure 1. Block Stacking  
Honeycomb Loss  
[DeMars, 1980]

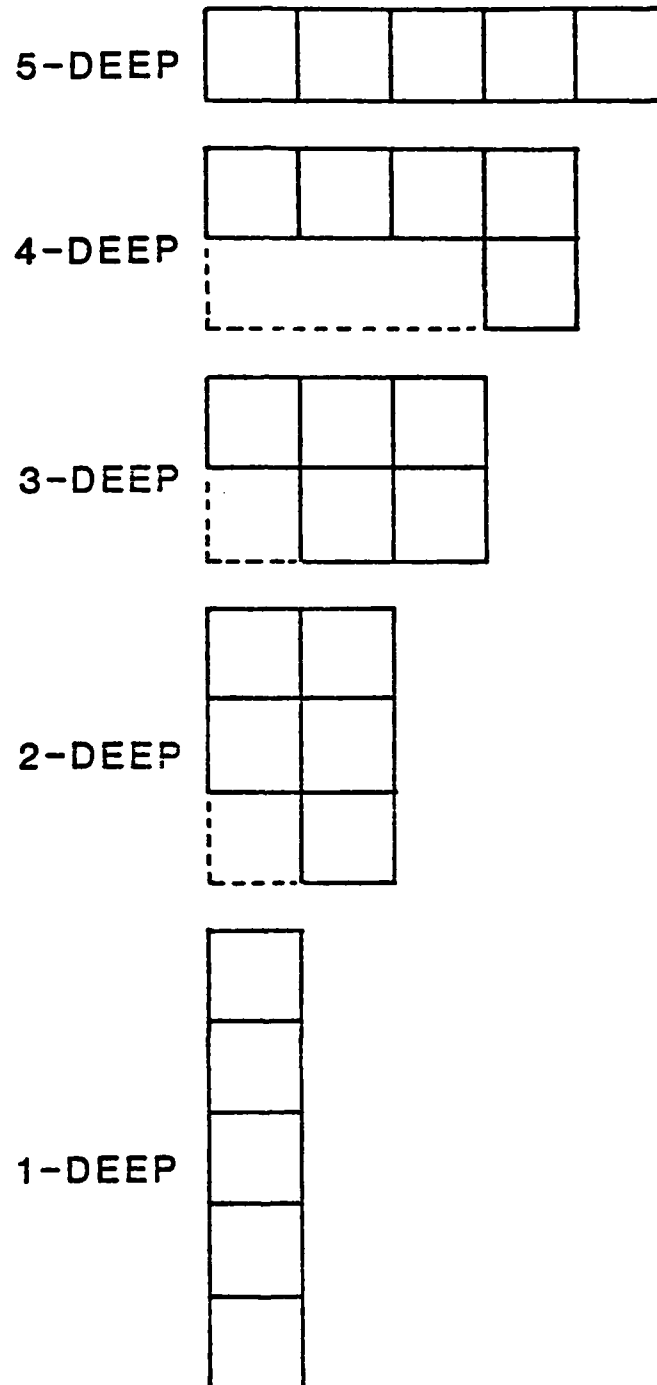


Figure 2. Storage Lane Depth Options [DeMars, 1980]

losses due to honeycombing.

The optimum lane depth is also influenced by handling times. Deeper lanes may require extra caution within the storage lane and thus greater handling times and costs.

#### Other Storage Methods

The analysis of block stacking storage can provide insight for the analysis of other storage alternatives. For example, single-deep pallet racks typically require more aisle space per pallet stored and have a greater equipment cost than block stacking. However, lot rotation and selectivity are facilitated, and honeycomb loss does not exist for single-deep racks. Double-deep racks and deep lane storage systems have horizontal but not vertical honeycomb loss. Drive-in racks are similar to block stacking, and both vertical and horizontal honeycombing can occur.

Flow racks are similar to deep lane storage systems with the exception that vacant spaces occur behind, rather than in front of, the lot during the withdrawal period. Since flow racks are served by both a storage aisle and a retrieval aisle, new stock from a different lot of the given product can be stored in the same lane with the existing lot. Also FIFO rotation exists with flow racks.

Block stacking is generally used in combination with lift trucks for storage and retrieval of stock. Storage systems for bulk items such as large coils often employ stacking with an overhead crane for stock movement. Such a system is subject to vertical honeycomb loss only.

In evaluating different storage methods, costs for space, labor, and equipment must be considered. Previous studies have investigated the space and handling aspects of storage systems design. The following section provides a summary of previous research in the area of block stacking storage

systems design.

### Related Literature

A survey of the literature reveals that little research has been directed toward design optimization for block stacking and deep lane storage methods. However, several references provide general definitions or offer guidelines for the design of block stacking storage systems. Among these are Jenkins [1968], Falconer and Drury [1975], Hulett [1970], and Apple [1972].

Apple [1972] discussed block stacking and the problem of honeycomb loss. However, a method for designing block stacking storage systems was not provided.

Jenkins [1968] provided a procedure for setting "space utilization standards" to control floor space costs in the warehouse. Although the procedure required the determination of the optimum layout before setting standards, Jenkins did not discuss the problem of determining optimum layouts. In planning aisle widths for block stacking, Jenkins suggested that aisles could be more narrow than those required for rack storage; this suggestion was based on the supposition that all storage lanes are not fully occupied at any given time (as a result of stock withdrawal and honeycombing).

Falconer and Drury [1975] compiled an architect's guide for the design of industrial storage and distribution systems and buildings. They provided comparisons of space requirements for block stacking and other storage methods. However, the comparisons reflected only the maximum inventory requirements. For storage stacks up to four pallet loads high and for any lane depth, they reported that storage blocks longer than twenty

storage lanes gain little in space savings. Hulett [1970] and Falconer and Drury provided a rule of thumb for estimating maximum block stacking space requirements at storage depths of two to three pallet stacks. The rule is: to determine total space requirements, add an allowance of thirty-four percent for aisle space to the area required for pallets.

A Warehouse Modernization and Layout Planning Guide [Department of the Navy, 1978] provides a discussion of block stacking and the problem of honeycombing. The guide reports that the lift truck operator may have difficulty in maneuvering when stacks are more than four pallets high or when storage lanes are deeper than two vehicle lengths. The guide also examines the aisle widths and clearances required for block stacking single-deep racks, and double-deep racks for a variety of lift trucks. Elemental times for storage or retrieval of pallets in racks are given for various lift truck types and load heights. Both initial costs and operating costs are given for pallet handling equipment including lift trucks and S/R machines.

Others including Thornton [1961], Moder and Thornton [1965], Hemmi [1964], Kind [1965, 1975], Berry [1968], Roberts [1968], and White [1970] have directed their efforts toward block stacking design optimization. However, most have not considered the storage-retrieval distribution or the problem of honeycomb loss.

Thornton [1961] and Moder and Thornton [1965] investigated block stacking space efficiency and the effects of (1) clearances between storage lanes (in excess of required operating clearances) and of (2) the angle of the storage lanes with respect to the aisle. The studies examined space efficiency, defined as the ratio of usable storage space to total space. However, all storage positions in the storage lane were considered usable space. Space efficiency was calculated for fully occupied storage lanes.

Thus, any FIFO requirements and the possibility of honeycomb losses were neglected. Moder and Thornton developed mathematical models to determine the optimum values for clearances, lane depths, and angles for various lift truck and pallet specifications. Their findings included the following.

1. The optimum excess clearance between lanes is zero.
2. The optimum angle is obtained with the storage lane perpendicular to the aisle.
3. Lane depth should be as large as possible; however, no significant gains in space efficiency result from choosing storage lanes depths greater than five pallet stacks.

The studies did not consider the effects of clearances and angle stacking on handling requirements.

As part of a larger study on the influence of layout on storage space and handling costs, Hemmi [1964] investigated the effect of stacking depth on space efficiency, assuming no FIFO requirements. Eighteen "representative" warehouse layout patterns and sizes were analyzed for lane depths of one to six pallet stacks. Since Hemmi used the same space efficiency measure Moder and Thornton used, he similarly concluded that space efficiency was greater for deeper storage lanes. In his investigation of the effects of layout and warehouse size, Hemmi determined that

- layout significantly affects space efficiency only for small warehouse sizes, and
- handling times are more important than space efficiency as a design consideration for larger warehouses.

Berry [1968] developed total cost expressions including space requirements costs and costs for the average distance travelled by all goods, given the maximum stock volume. He assumed that each product required two storage lanes (one for storage and one for retrieval) which on average

would be half filled. Lot sizes were not considered, and only one lane depth was allowed. Distance expressions for lift truck travel assumed the same cost for both vertical and horizontal movement. Berry found that the optimum stack depth was greater for fast moving product lines than for slow moving lines. He also concluded that the layout for maximum space utilization differed from the layout for minimum handling distance.

Roberts [1968] examined the optimum size of a storage block including lane depth (which he referred to as "storage block depth") and the number of lanes required ("storage block length"). He first investigated the case of one storage lane per product and then extended the problem to consider "lot splitting," i.e. more than one storage lane per product. Both dynamic programming and integer programming were used to determine the block lengths and depths for minimum space requirements. Although Roberts identified the problem of honeycombing, his analysis did not reflect space losses over the life of the product lot. Roberts also examined layout issues and used dynamic programming to determine design configurations for minimum handling and perimeter costs.

White [1970] used extreme point ranking, branch and bound, and dynamic programming techniques to solve several segregated storage and warehouse sizing problems. He investigated the optimum distribution of products among compartments in an existing warehouse and the optimum number and sizes of compartments for a new warehouse. Models were developed for four cases of storage demand. However, the analysis did not include consideration of honeycomb losses due to product withdrawal.

Kind [1965, 1975] based his study of block stacking on the argument that the measure of space utilization should reflect the average space requirements over the life of the product lot. He defined space

utilization as the ratio of cumulative space occupied over the life of the product lot to cumulative space committed over the life of the lot. Kind recommended an approximate expression for determining the optimum storage lane depth for a product. This expression is given by:

$$\text{OPTIMUM LANE DEPTH} = \sqrt{\frac{(\text{LOT SIZE})(\text{AISLE WIDTH})}{\text{STACK HEIGHT}}} - \frac{\text{AISLE WIDTH}}{2}$$

where lane depth, lot size, aisle width, and stack height are measured in pallet loads. Although he did not provide the derivation, Kind reported that the approximation was developed "using operations research techniques."

A simulation approach was used by Marsh [1978, 1979] to study space utilization for alternative block stacking policies. Three operating policies were analyzed.

1. Straight queueing - A shipment (product lot) to be stored waits for an available lane of the desired depth.
2. Upward product set search - A shipment "seeks" storage in a lane of greater storage depth when a lane of the desired depth is not available.
3. Downward product set search - A shipment "seeks" storage in a lane of less storage depth when a lane of the desired depth is not available.

For a specific set of products with given lot sizes, reorder points, and lead times, Marsh determined lane depth requirements for the cases of splitting lots into one, two, or three lanes per lot. He then arbitrarily chose six lane depths and assigned a lot splitting rule to each product.

Marsh's simulation was designed to analyze the operational aspects of block stacking, including the effects of honeycombing on space utilization. Specifically, he examined the effect on space utilization of storing in longer or shorter lanes when all lanes of the assigned depth were fully

occupied. For his particular product set, Marsh determined that a policy of storage in deeper lanes was preferable.

However, Marsh's results were based on a comparison of policies for some arbitrary design decisions for lane depths and lot splitting. His results might have been quite different if his policy comparisons had been based on optimum design decisions.

With the exception of Kind's studies, previous block stacking research has not considered the effect of the storage-retrieval distribution on space utilization. The following section presents some space requirements models which do reflect various storage-retrieval distributions.

#### Space Requirements Models

A common objective of warehouse planners is to maximize space utilization. An alternate and consistent objective is to minimize the average space required. Much of the previous research in this area has focused on space utilization or space requirements at the maximum inventory level. The models presented here reflect average space requirements over the life of the product lot, including aisle space and clearances.

#### Block Stacking

Additional assumptions and restrictions are required for the analysis of block stacking storage systems. First, products are stored in back-to-back lanes as illustrated in Figure 3. Thus each storage lane can be "charged" with one-half of the aisle space in front of the lane. In this analysis, all storage lanes for a product lot are restricted to be of equal depth. Also, for a given product lot, units from any partially filled lane must be withdrawn before withdrawal from a full lane. For a single product, storage lane depth is the decision variable. It is assumed that product

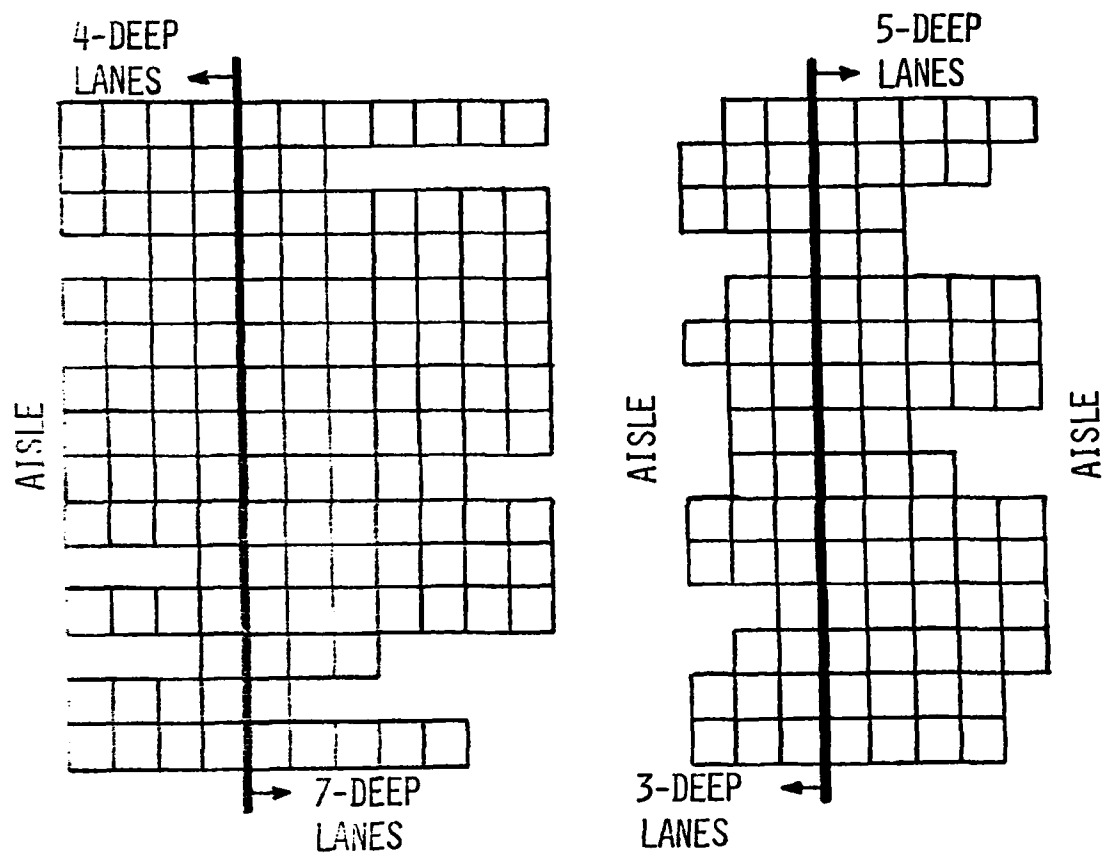


Figure 3. Block Stacking Storage Lanes

or building constraints limit stacking height to some known integer quantity of pallet loads.

Influence of the storage-retrieval distribution. The total space required by a product lot changes over time as loads are added to or withdrawn from storage. When a storage lane is occupied by one or more pallet loads of a given lot during the withdrawal period, that lane is unavailable for storage of any other product or lot. Although unused storage positions may exist within the lane, the positions must remain vacant as honeycomb losses to achieve FIFO lot rotation. Thus the entire storage lane and its associated aisle space are committed for storage of the product lot until all pallet loads are withdrawn from the lane. The lane is then free for storage of any other product or lot. A product's storage-retrieval distribution determines when storage lanes are required and when they are freed.

The first block stacking problem to be examined assumes the Case 1 storage-retrieval distribution illustrated in Figure 4. Characteristics of this distribution include:

- instantaneous replenishment of the product lot,
- uniform withdrawal rate, and
- a withdrawal size of one pallet load.

Model description. Given the storage-retrieval assumptions described above, an expression can be developed for the average floor space required for block stacking over the life of the product lot. This expression, given by DeMars, Matson, and White [1981], is:

$$S = \frac{y(W + c)(0.5A + xL)(2Q - xyT + xT)}{288Q}$$

where

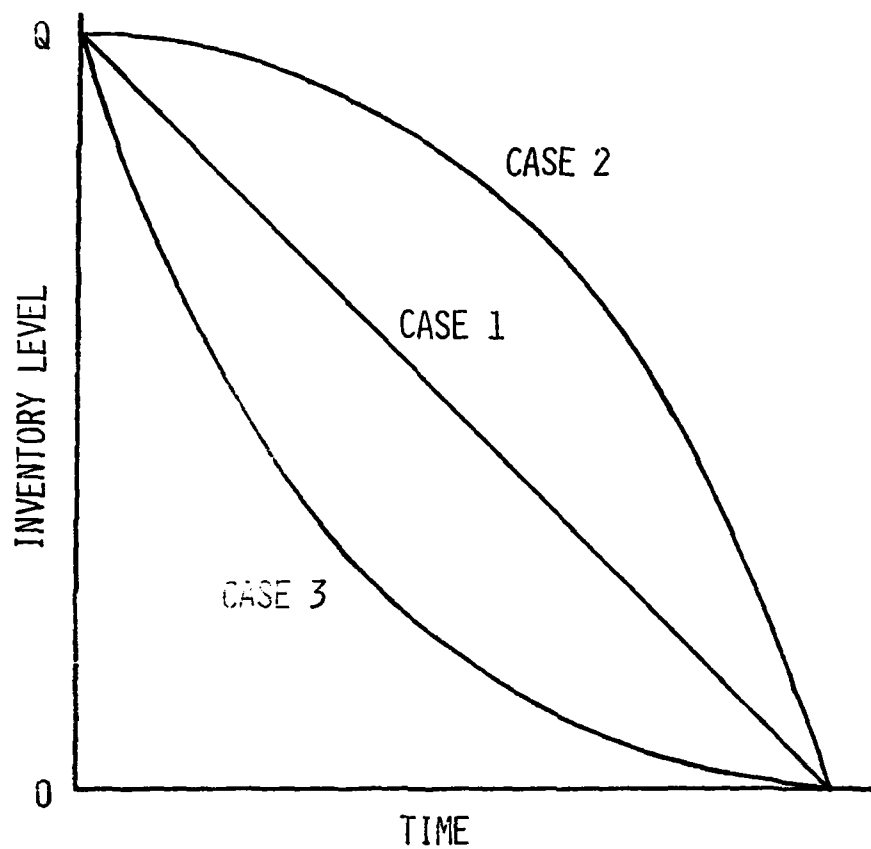


Figure 4. Storage-Retrieval Distributions for  
Three Alternative Withdrawal Rates  
[DeMars, Matson, and White, 1981]

$S$  = average amount of floor space required for block stacking (ft.<sup>2</sup>)

$x$  = depth of storage lane (integer number of pallet stacks)

$y$  = number of storage lanes for full lot storage (integer number of pallets)

$Q$  = lot size (integer number of pallets)

$L$  = pallet length or depth (inches)

$W$  = pallet width (inches)

$h$  = load height, including pallet (inches)

$c$  = clearance between lanes (inches)

$T$  = number of storage tiers or levels (integer number of pallets)

$A$  = aisle width (inches).

This expression also assumes that  $y$  is defined as the smallest integer greater than or equal to  $Q/xT$ . That is

$$y = \left\lceil \frac{Q}{xT} \right\rceil$$

where  $\lceil f(x) \rceil$  = the smallest integer greater than or equal to  $f(x)$ .

Figure 5 illustrates dimensions of floor stacks in block stacking.

To determine the optimum lane depth  $x$  for minimum average space requirements, the following problem must be solved [DeMars, Matson, and White 1981]:

$$\text{Minimize } S = \frac{y(W + c)(0.5A + xL)(2Q - xyT + xT)}{288Q}$$

$$\text{subject to } xyT \geq Q$$

The constraint is required to insure that only those designs which provide sufficient storage space for the lot size are considered. The single decision variable is  $x$  since  $y$  is defined as  $\left\lceil \frac{Q}{xT} \right\rceil$ .

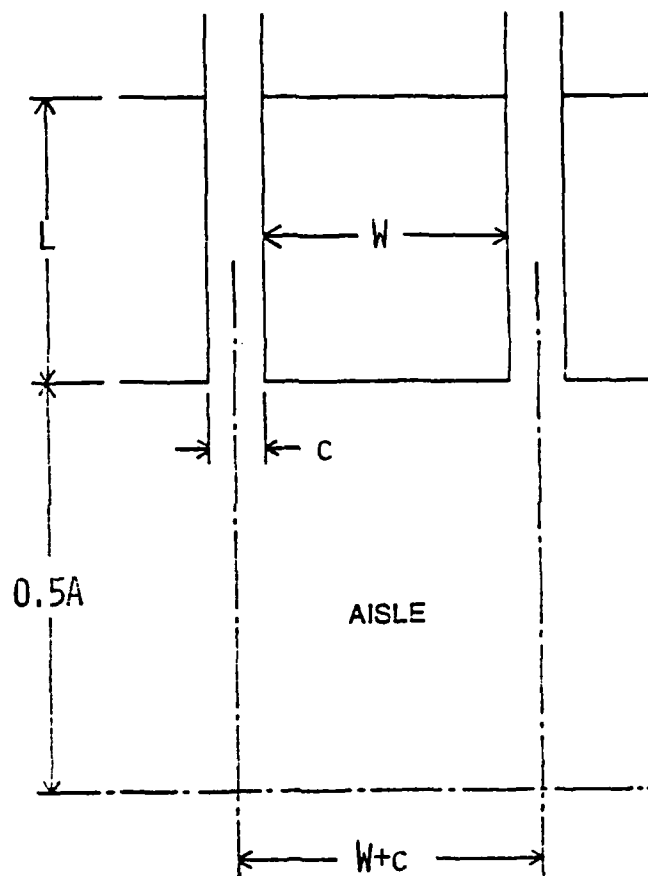


Figure 5. Floor Stacks for Block Stacking [DeMars, 1980]

It should be noted that this optimization model minimizes average floor space requirements rather than cubic space requirements. For purposes of this analysis, it is assumed that storage height is not a decision variable. Such an assumption would be valid for existing buildings. Also, when the product stored has a stacking limit, due to load crushing, its limit is typically less than the height of a conventional warehouse. However, the model can easily be modified to reflect average cubic space requirements rather than average floor space.

Solution procedure. An examination of the space requirements model for block stacking shows that the objective function is nonlinear with an integer decision variable. Further investigation reveals that the function is nonconvex with respect to  $x$  and is subject to the presence of local minima. Thus, to obtain a global minimum requires total enumeration over all feasible values of  $x$ . However, since the maximum feasible lane depth for most storage situations is less than or equal to thirty pallet stacks, solution by total enumeration will generally require at most thirty calculations of the objective function.

An example of calculations for a product with lot size of fifteen is given in Table 1. In this example, stacking height is three pallet loads, load dimensions are 50" x 42" x 48", clearance between lanes is 10", and the aisle is 144" wide.

Model derivation. Since withdrawal from storage is uniform at the rate of one pallet load per withdrawal, there are  $Q$  inventory states over the life of the product lot. Each of these states  $\{Q, Q-1, Q-2, \dots, 2, 1\}$  has an equal probability of occurrence. Average space requirements are

Table 1. Block Stacking Example: Case 1

$$\begin{array}{lll}
 Q = 15 & L = 50 & \text{Min } S = \frac{52y(72 + 50x)(30 - 3xy + 3x)}{4320} \\
 T = 3 & W = 42 & \text{s.t. } xy \geq 5 \\
 & c = 10 & \\
 & A = 144 & 
 \end{array}$$

x	y	S	
1	5	132.17	
2	3	111.80	OPTIMUM
3	2	112.23	
4	2	117.87	
5	1	116.28	

determined by considering the floor space required for each inventory state and the probability of that state.

As an example, consider the problem defined in Table 1 and the optimum solution to the problem. Table 2 shows, for the optimum lane depth ( $x = 2$ ), how the space requirements vary as the lot is depleted.

Continuing the example, Figure 6 illustrates space utilization over the life of the lot. Space utilization at a given inventory level is defined as the ratio of floor space required by the pallet loads, not including aisle space or clearances, to total floor space committed for that inventory level. As Figure 6 indicates, space utilization increases when a storage lane is freed.

The characteristics of the block stacking operation can be expressed in terms of the model variables and parameters. The space required for each committed storage lane is an area  $(W + c)$  inches wide by  $(0.5A + xL)$

Table 2. Space Requirements for Each Inventory State

<div> <div>Q = 15</div> <div>L = 50</div> <div>T = 3</div> <div>W = 42</div> <div>x = 2</div> <div>c = 10</div> <div>A = 144</div> </div>				
(1) Inventory Level	(2) Number of Lanes Required	(3) Total Space Required (ft. <sup>2</sup> )	(4) Probability of Inventory Level	(3)(4)
15	3	186.33	1/15	12.422
14	3	186.33	1/15	12.422
13	3	186.33	1/15	12.422
12	2	124.22	1/15	8.281
11	2	124.22	1/15	8.281
10	2	124.22	1/15	8.281
9	2	124.22	1/15	8.281
8	2	124.22	1/15	8.281
7	2	124.22	1/15	8.281
6	1	62.11	1/15	4.141
5	1	62.11	1/15	4.141
4	1	62.11	1/15	4.141
3	1	62.11	1/15	4.141
2	1	62.11	1/15	4.141
1	1	62.11	1/15	4.141
AVG = 111.798				

inches deep. For any (x, y) pair which satisfies the constraint  $xyT \geq Q$ , a "snapshot" of the block stacking design at maximum inventory level Q reveals the following:

- (y - 1) lanes are fully occupied with xT pallet loads in each lane,

$$\text{UTILIZATION} = \frac{I_t WL}{T(W + c)(0.5A + xL) \left[ \frac{I_t}{xT} \right]}$$

where  $I_t$  = inventory level at time  $t$

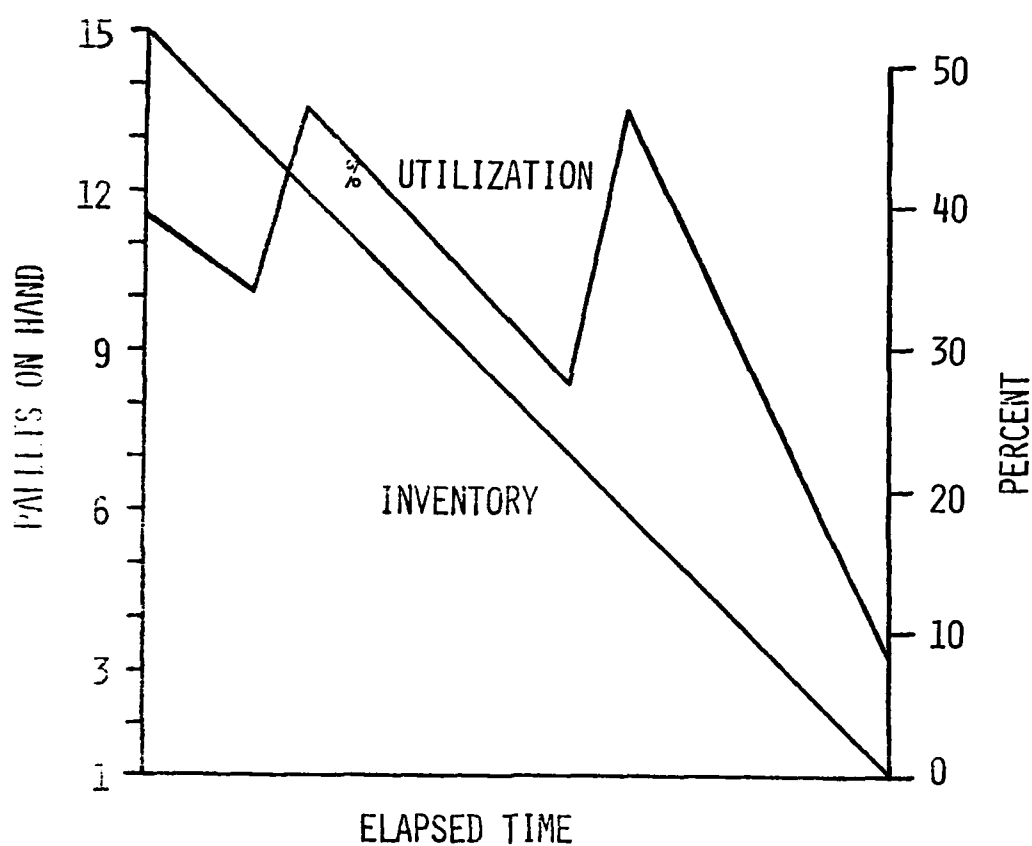


Figure 6. Floor Space Utilization (Adapted from [DeMars, 1980])

- one lane is partially or fully occupied, containing  $[Q - x(y - 1)T]$  pallet loads.

An examination of the storage lanes committed to the lot over the life of the lot shows that

- $y$  lanes are committed until  $[Q - x(y - 1)T]$  loads are withdrawn
- $(y - 1)$  lanes are committed until an additional  $xT$  loads are withdrawn
- $(y - 2)$  lanes are committed until an additional  $xT$  loads are withdrawn
- $\vdots$
- 1 lane is committed until the remaining  $xT$  loads are withdrawn.

The average number of storage lanes required,  $\bar{n}$ , is thus given by

$$\bar{n} = \left[ \sum_{k=1}^{y-1} (y - k) \frac{xT}{Q} \right] + \frac{y[Q - x(y - 1)T]}{Q}$$

which reduces to

$$\bar{n} = \frac{y(2Q - xyT + xT)}{2Q}$$

Therefore, the average floor space requirements are given by

$$S = \frac{y(A + c)(0.5A + xL)(2Q - xyT + xT)}{288Q}$$

which is the expression for space requirements presented earlier.

Other storage-retrieval distributions. Two other storage-retrieval distributions are illustrated in Figure 4. Case 2 assumes an increasing (geometric) rate of stock withdrawal, whereas Case 3 assumes a decreasing rate of withdrawal. Table 3 illustrates the inventory levels and the corresponding time in stock for the three cases.

In developing the space requirements models for Case 2 and Case 3, it is necessary to determine the average number of storage lanes required over the life of the product lot. For Case 2, the average is

Table 3. Inventory States for Three Withdrawal Rates

Inventory Level	Time in Stock		
	Case 1	Case 2	Case 3
Q	p	$p^0$	$p^{Q-1}$
Q-1	p	$p^1$	$p^{Q-2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Q-k	p	$p^k$	$p^{Q-k-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	p	$p^{Q-2}$	$p^1$
1	p	$p^{Q-1}$	$p^0$

$$\eta_2 = \frac{1}{(1 - p^Q)} \left[ y - p^{Q-xT(y-1)} \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) \right]$$

For Case 3, the average is

$$\eta_3 = \frac{1}{(1 - p^Q)} \left[ \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) - yp^Q \right]$$

(The averages,  $\eta_1$ ,  $\eta_2$ , and  $\eta_3$ , are derived in the Appendix.) As before, each storage lane is  $(W + c)$  inches wide by  $(0.5A + xL)$  inches deep. Therefore, for Cases 2 and 3, the average floor space to be minimized is given by

$$S_2 = \frac{(W + c)(0.5A + xL)}{144(1 - p^Q)} \left[ y - p^{Q-xT(y-1)} \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) \right], \quad \text{CASE 2}$$

$$S_3 = \frac{(W + c)(0.5A + xL)}{144(1 - p^Q)} \left[ \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) - yp^Q \right], \quad \text{CASE 3}$$

where

$$0 < p < 1$$

and  $S_k$  = space requirements for Case k. Table 4 illustrates the results obtained for the three different storage-retrieval assumptions. As shown, Case 2 will generally require deeper lanes than Case 1 and Case 3.

Table 4. Block Stacking Example: Cases 1, 2, and 3				
<p>Q = 15      L = 50</p> <p>T = 3      W = 42</p> <p>p = 0.8      c = 10</p> <p>A = 144</p>				
x	y	$S_1$	$S_2$	$S_3$
1	5	132.17	182.14	82.30
2	3	111.80	149.30	78.94
3	2	112.23	141.50	88.42
4	2	117.87	147.94	101.66
5	1	116.28	116.28	116.28

#### Other Storage Alternatives

Single-deep racks. The warehouse planner may wish to compare the space requirements for block stacking versus single-deep pallet rack space requirements. Although the use of single-deep racks eliminates honeycomb loss, additional space is required for vertical and horizontal rack members, as shown in Figure 7. For the storage-retrieval distributions of Figure 4,

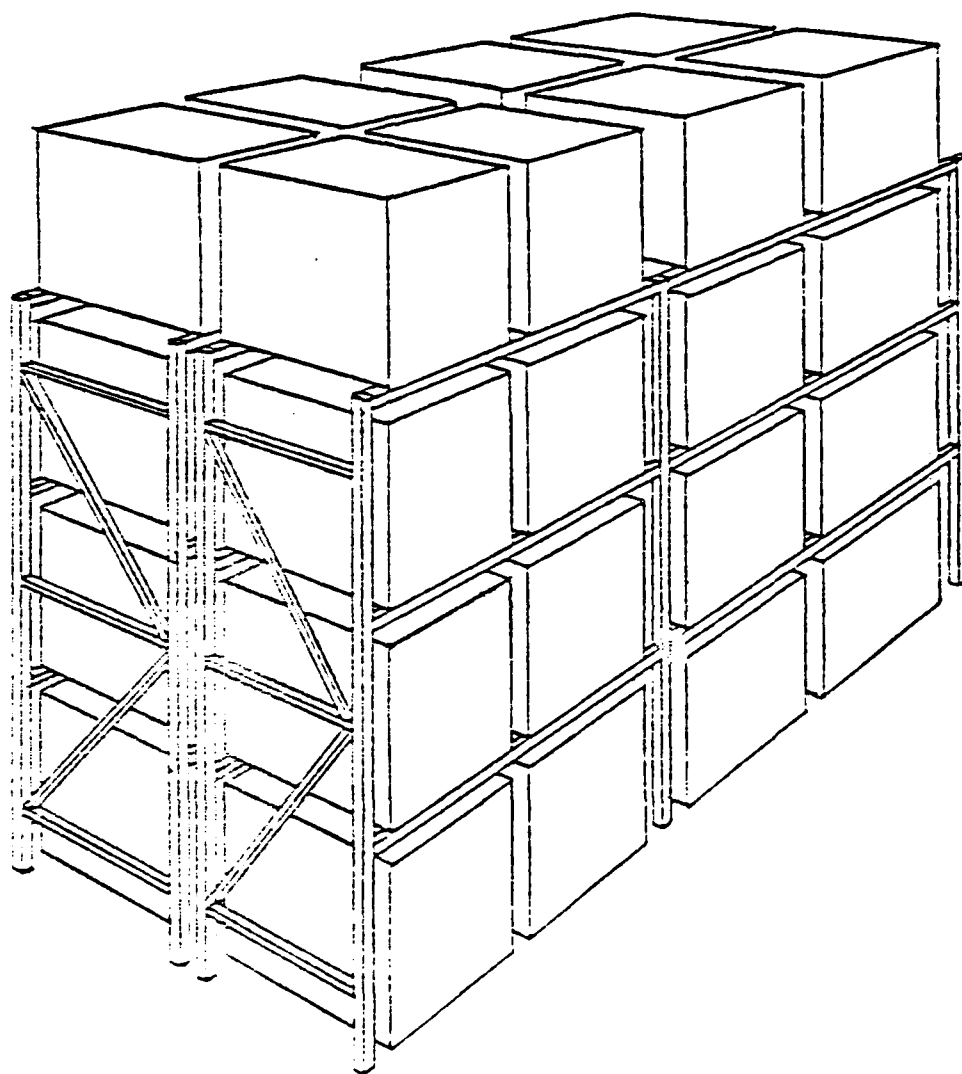


Figure 7. Rack Space Allowances [DeMars, 1980]

the average space requirements for single-deep rack storage are

$$S_1 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L](Q + 1)}{288T}$$

$$S_2 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L]}{144T(1 - p^Q)} \left[ Q - p \left( \frac{1 - p^Q}{1 - p} \right) \right]$$

$$S_3 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^Q}{1 - p} \right) - Qp^Q \right]$$

where

$f$  = flue spacing, load-to-load (inches)

$c$  = side-to-side clearance between loads and vertical rack member (inches)

$r$  = width of vertical rack member (inches)

$v$  = maximum number of storage slots required

Other notation is consistent with that used previously for the block stacking models. These clearances and dimensions are illustrated in Figure 8.

Note that these space requirements expressions are based on the assumption that two pallet loads are stored side-by-side in each rack opening.

For a product with  $Q = 15$ ,  $T = 4$ ,  $L = 50"$ ,  $W = 42"$ ,  $c = 4"$ ,  $f = 6"$ , and  $r = 3"$ , the average space requirements for single-deep rack storage are as follows:

Case 1	85.94 ft. <sup>2</sup>
Case 2	124.04 ft. <sup>2</sup>
Case 3	47.83 ft. <sup>2</sup>

The development of space requirements expressions for single-deep racks follows the procedures used in formulating the block stacking models. Derivations for the average number of storage slots for each of the three

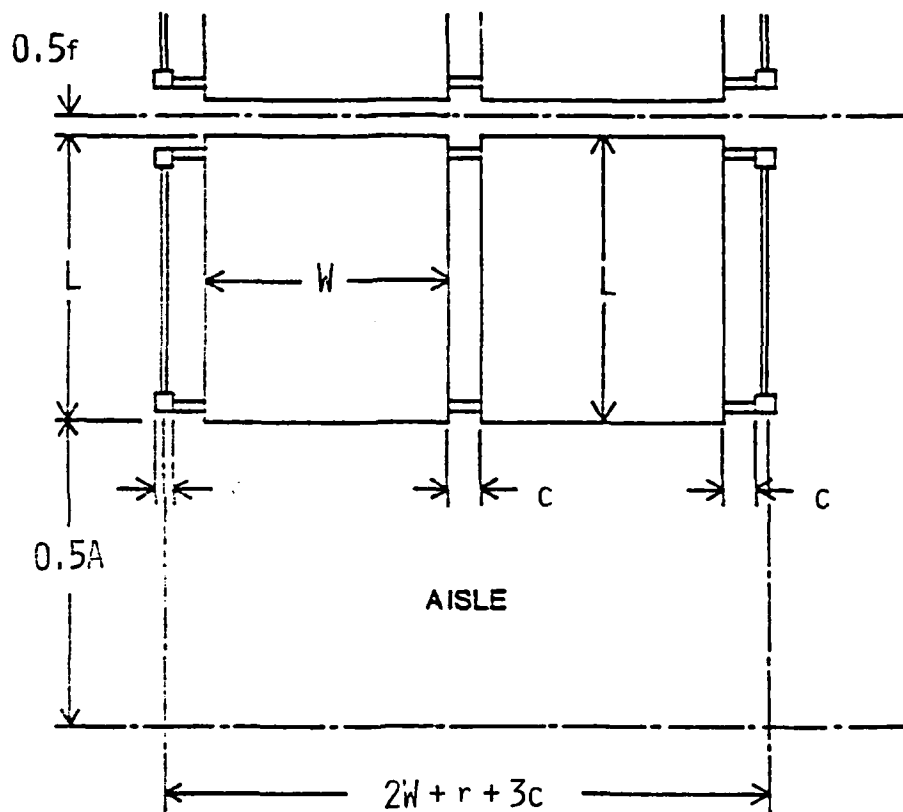


Figure 8. Single-Deep Racks [DeMars, 1980]

cases are illustrated in the Appendix. Each pallet load requires a storage slot which is  $(W + 0.5r + 1.5c)$  inches wide by  $[0.5(A + f) + L]$  inches deep. For single-deep racks,  $x = 1$  and  $v = Q$ . The average number of storage slots is divided by  $T$  storage levels.

Double-deep racks. Double-deep racks permit horizontal but not vertical honeycombing. The average space requirements are determined by the following expressions for the three storage-retrieval distributions:

$$S_1 = \frac{v(W + 0.5r + 1.5c)[0.5(A + f) + 2L](Q + 1 - v)}{144QT}$$

$$S_2 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + 2L]}{144T(1 - p^Q)} \left[ v - p^{Q-2(v-1)} \left( \frac{1 - p^{2v}}{1 - p^2} \right) \right]$$

$$S_3 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + 2L]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^{2v}}{1 - p^2} \right) - vp^Q \right]$$

$$\text{where } v = \begin{cases} \frac{Q}{2}, & Q \text{ even} \\ \frac{Q+1}{2}, & Q \text{ odd} \end{cases}$$

$$\text{or } v = \frac{2Q + 1 - (-1)^Q}{4}$$

The average number of storage slots for each case is derived in the Appendix.

Figure 9 illustrates double-deep rack storage. For the example  $Q = 15$ ,  $T = 4$ ,  $L = 50''$ ,  $W = 42''$ ,  $c = 4''$ ,  $f = 6''$ , and  $r = 3''$ , the average space requirements for double-deep rack storage are as follows:

Case 1	64.17 ft. <sup>2</sup>
Case 2	91.04 ft. <sup>2</sup>
Case 3	37.69 ft. <sup>2</sup>

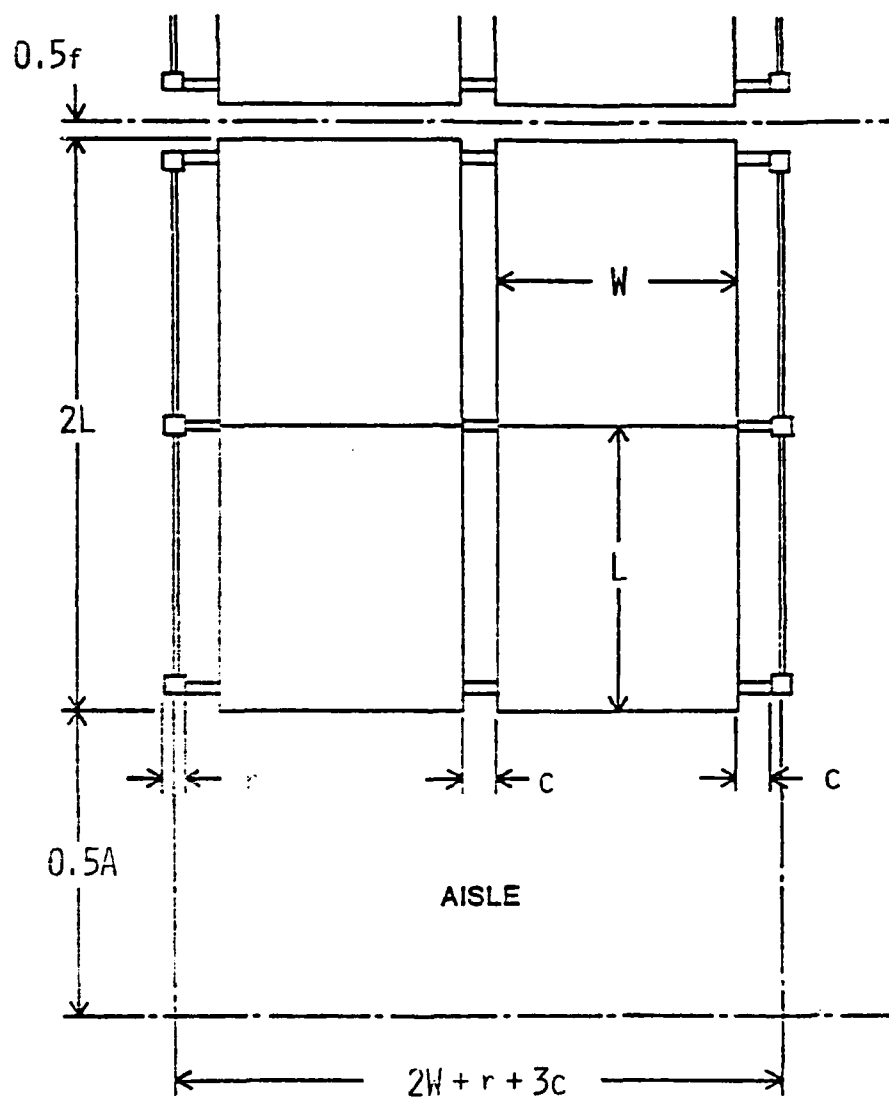


Figure 9. Double-Deep Racks [DeMars, 1980]

Deep lane storage systems. Like double-deep racks, deep lane storage systems permit horizontal, but not vertical, honeycomb loss. The storage depth for minimum average space requirements can be determined by minimizing the following expressions for the three storage-retrieval cases.

$$S_1 = \frac{v(W + r + 2c)[0.5(A + f) + xL](2Q - xv + x)}{288QT}$$

$$S_2 = \frac{(W + r + 2c)[0.5(A + f) + xL]}{144T(1 - p^Q)} \left[ v - p^{Q-x(v-1)} \left( \frac{1 - p^{xv}}{1 - p^x} \right) \right]$$

$$S_3 = \frac{(W + r + 2c)[0.5(A + f) + xL]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^{xv}}{1 - p^x} \right) - vp^Q \right]$$

Figure 10 illustrates deep lane storage system dimensions.

This formulation follows from the block stacking model. A storage slot for a deep lane system has a width of  $(W + r + 2c)$  inches and a depth of  $[0.5(A + f) + xL]$  inches. At peak inventory  $(v - 1)$  storage slots are fully occupied with  $x$  pallet loads per storage slot; one slot is partially or fully occupied with  $[Q - x(v - 1)]$  loads. The average number of storage slots required for each case is derived in the Appendix.

Again, total enumeration is used to determine the global optimum.

Table 5 provides an example problem.

Summary. Table 6 provides a comparison of the space requirements for block stacking, single-deep racks, double-deep racks, and deep lane storage system for the storage-retrieval distributions discussed previously. Pallets are stacked three high for block stacking and four high for the other methods.

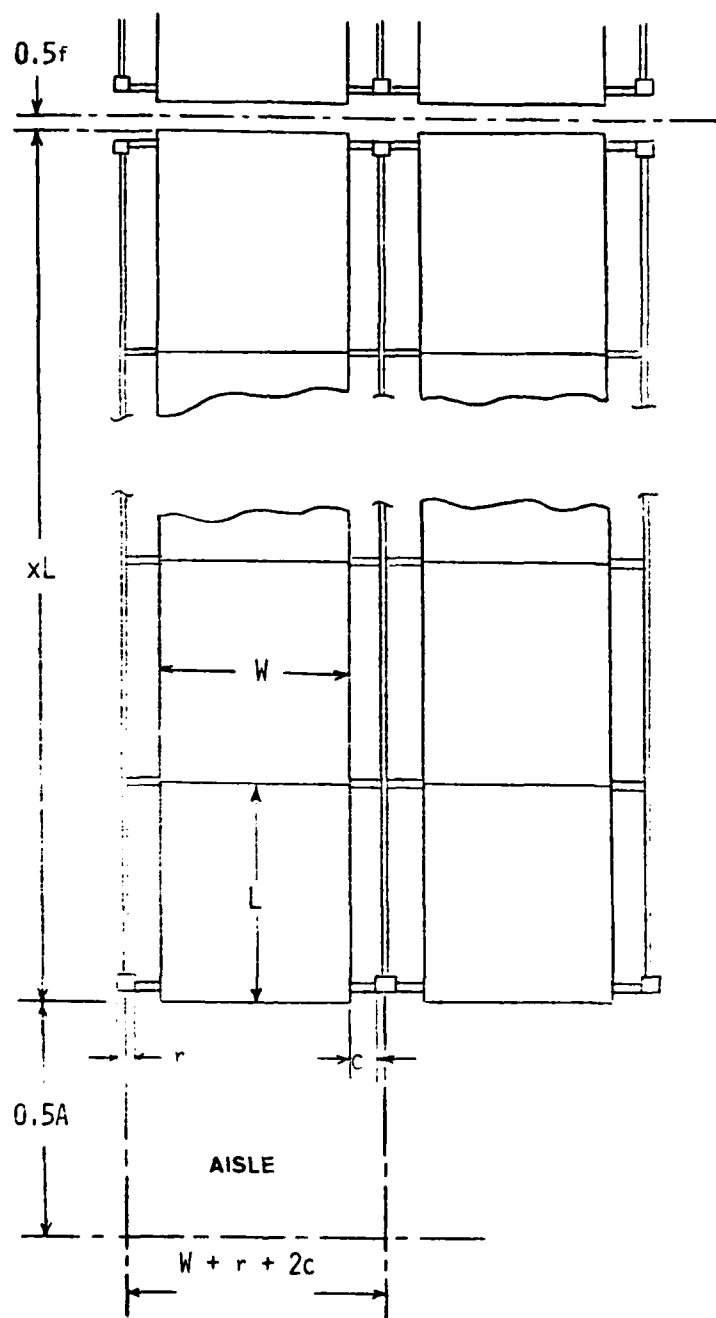


Figure 10. Deep Lane Storage

Table 5. Deep Lane Storage System Example

$Q = 15$ $L = 50''$ $T = 4$ $W = 42''$ $c = 3''$ $p = 0.8$ $f = 6''$ $r = 3''$				
x	v	$S_1$	$S_2$	$S_3$
1	15	88.54	127.80	49.28
2	8	66.11	91.79	41.35
3	5	59.77	82.34	37.19
4	4	58.44	79.68	37.99
5	3	57.55	75.45	39.65
6	3	59.77	79.79	42.17
7	3	60.21	77.88	44.79
8	2	61.68	76.50	47.85
9	2	65.08	82.04	51.25
10	2	67.88	86.39	54.70
11	2	70.10	89.20	58.25
12	2	71.72	89.99	61.86
13	2	72.75	88.14	65.51
14	2	73.19	82.84	69.25
15	1	73.05	73.05	73.05

#### Other Storage-Retrieval Distributions

The block stacking models presented previously were based on the assumptions of instantaneous replenishment of stock and of a withdrawal size of one pallet load for three different withdrawal rates. If  $w$  is defined as a constant integer withdrawal size ( $w \geq 1$ ) and consideration is limited to a uniform rate of withdrawal, other models can be developed to

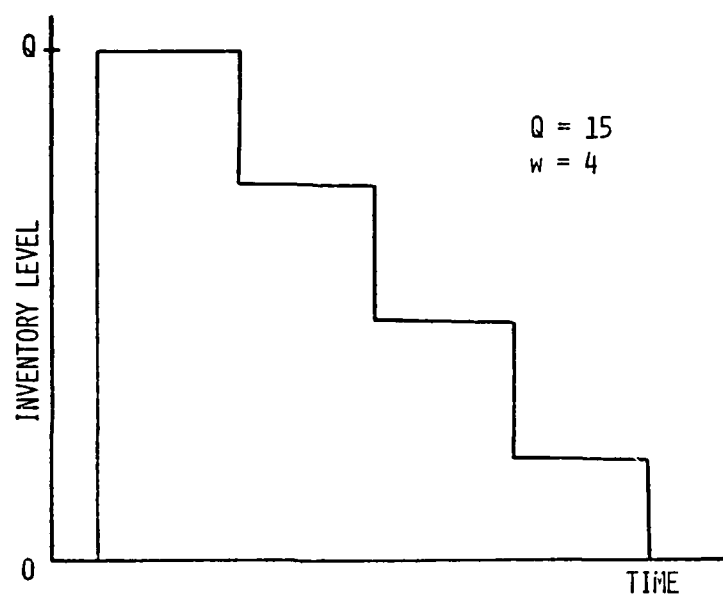
Table 6. Space Requirements Summary

$Q = 15$ $L = 50''$ $p = 0.8$ $W = 42''$ $A = 144''$				
Method	Block Stacking	Single-Deep Racks	Double-Deep Racks	Deep Lane System
T	3	4	4	4
c	10	4	4	3
h	---	3	3	3
h	---	6	6	6
$S_1$	111.80	85.94	64.17	57.55
n	2	---	---	5
$S_2$	116.25	124.04	91.04	73.05
n	5	---	---	15
$S_3$	78.94	47.83	37.69	37.19
n	2	---	---	3

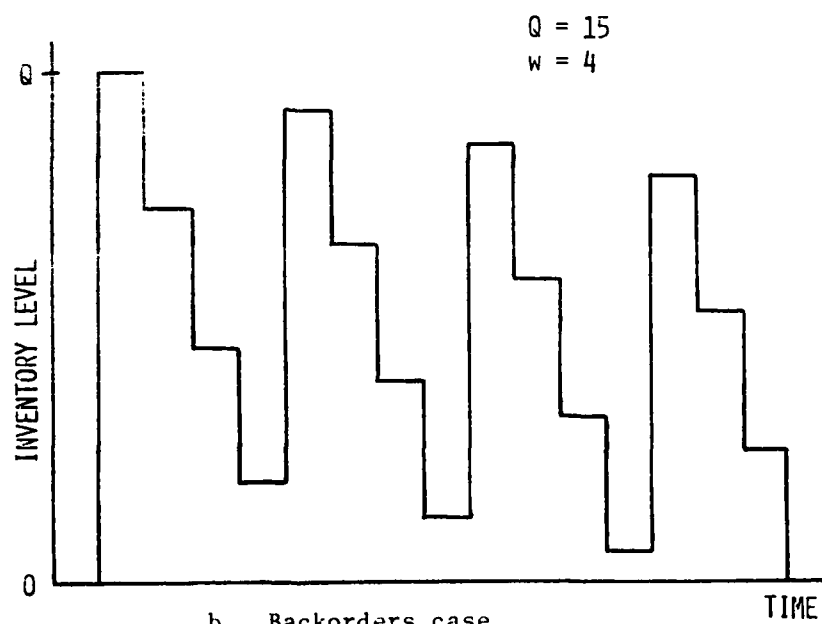
determine the optimum design for a withdrawal size greater than one pallet load. Two cases must be considered:

1. Lost sales
2. Backorders

(1) The lost sales case is illustrated in Figure 11(a). When the number of units in stock for a particular lot falls to  $w$  loads or less, it



a. Lost sales case



b. Backorders case

Figure 11. Storage-Retrieval Distributions for Uniform Withdrawal in Steps

is assumed that the next order for  $w$  loads is filled by the remainder of the lot. The order cannot be satisfied by a different lot, and any unsatisfied demands are treated as lost sales. The optimization problem for this case is given by

$$\text{Minimize } S = \frac{(W + c)(0.5A + xL)}{144m} \sum_{i=1}^m \left\lceil \frac{Q - w(i-1)}{xT} \right\rceil$$

where

$w$  = withdrawal size (integer number of pallet loads)

$m = \left\lceil \frac{Q}{w} \right\rceil$ , number of withdrawals required to deplete the storage lot

There are  $m$  inventory states  $\{Q, Q-w, Q-2w, \dots, Q-(m-1)w\}$ , and the average number of lanes required is given by

$$\frac{1}{m} \sum_{i=1}^m \left\lceil \frac{Q - w(i-1)}{xT} \right\rceil$$

Table 7 provides an example for a product with a lot size of 15 and a withdrawal size of 4.

(2) The backorders case is illustrated in Figure 11(b). It is assumed that unsatisfied demands result in backorders which are satisfied before storage of the incoming lot. For the backorders case, the storage-retrieval pattern forms a repeatable "period" of inventory states. If  $d$  is defined as the greatest common divisor of  $Q$  and  $w$ , i.e.  $d = \text{gcd}(Q, w)$ , there are thus  $\frac{Q}{d}$  unique inventory states within a period, and each period requires  $\frac{w}{d}$  replenishment-withdrawal cycles. Although the states may occur in a different sequence, the inventory states within a period are  $\{Q, Q-d, Q-2d, \dots, d\}$ . Table 8 provides an example of inventory states

Table 7. Block Stacking Example: Lost Sales Case

$$\begin{array}{ll}
 Q = 15 & L = 50'' \\
 T = 3 & W = 42'' \\
 w = 4 & c = 10'' \\
 & A = 144''
 \end{array}
 \quad m = \left\lceil \frac{Q}{w} \right\rceil = \left\lceil \frac{15}{4} \right\rceil = 4$$

x	S
1	143.18
2	124.22
3	120.25
4	122.78
5	116.28 OPTIMUM

within a period for a lot size of 15 and various withdrawal sizes.

Therefore, the average number of lanes required is

$$\frac{d}{Q} \sum_{i=1}^{Q/d} \left\lceil \frac{di}{xT} \right\rceil$$

The block stacking optimization model is given by

$$\text{Minimize } S = \frac{(W + c)(0.5A + xL)d}{144Q} \sum_{i=1}^{Q/d} \left\lceil \frac{di}{xT} \right\rceil$$

Table 9 provides an example for a lot size of 15 and  $w = 4$ .

#### Multiple Products

The optimum block stacking design for a single product can be determined by application of the appropriate model such as those presented

Table 8. Inventory States for Various Withdrawal Sizes  
(Backorders Case)

Q = 15		
<u>w</u>	<u>d</u>	<u>Inventory States</u>
1	1	{15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1}
2	1	{15, 13, 11, 9, 7, 5, 3, 1, 14, 12, 10, 8, 6, 4, 2}
3	3	{15, 12, 9, 6, 3}
4	1	{15, 11, 7, 3, 14, 10, 6, 2, 13, 9, 5, 1, 12, 8, 4}
5	5	{15, 10, 5}
6	3	{15, 9, 3, 12, 6}
7	1	{15, 8, 1, 9, 2, 10, 3, 11, 4, 12, 5, 13, 6, 14, 7}
8	1	{15, 7, 14, 6, 13, 5, 12, 4, 11, 3, 10, 2, 9, 1, 8}
9	3	{15, 6, 12, 3, 9}
10	5	{15, 5, 10}
11	1	{15, 4, 8, 12, 1, 5, 9, 13, 2, 6, 10, 14, 3, 7, 11}
12	3	{15, 3, 6, 9, 12}
13	1	{15, 2, 4, 6, 8, 10, 12, 14, 1, 3, 5, 7, 9, 11, 13}
14	1	{15, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}
15	15	{15}

previously. Extending the block stacking design problem to consider multiple products greatly increases the complexity of the problem. Some decisions required for the multi-product case include:

- the number of unique lane depths,
- values of the lane depths, and
- assignment of products to lane depths.

Table 9. Block Stacking Example: Backorders Case

Q = 15	L = 50"	d = gcd(15, 4) = 1
T = 3	W = 42"	
w = 4	c = 10"	
	A = 144"	

x	S	
1	132.17	
2	111.80	
3	112.23	OPTIMUM
4	117.87	
5	116.28	

Assuming that splitting of lots over different lane depths is not allowed, the number of lane depths could be varied from one depth for all products to  $n$  depths, i.e. one for each product, where  $n$  represents the number of different products requiring storage in the warehouse. If  $n$  different depths are allowed, the optimum set of lane depths can be obtained by solving  $n$  single product optimization problems. Some typical results for such a procedure are illustrated in Figure 12.

If only one lane depth is allowed for all products, the value of the optimum lane depth is found by minimizing the following expression:

$$S = \frac{(W + c)(0.5A + xL)}{144} \sum_{i=1}^n \sum_{j=1}^{Q_i} p_{ij} \left\lceil \frac{j}{xT_i} \right\rceil$$

where  $p_{ij}$  = probability of having  $j$  units of a product  $i$  lot on hand.

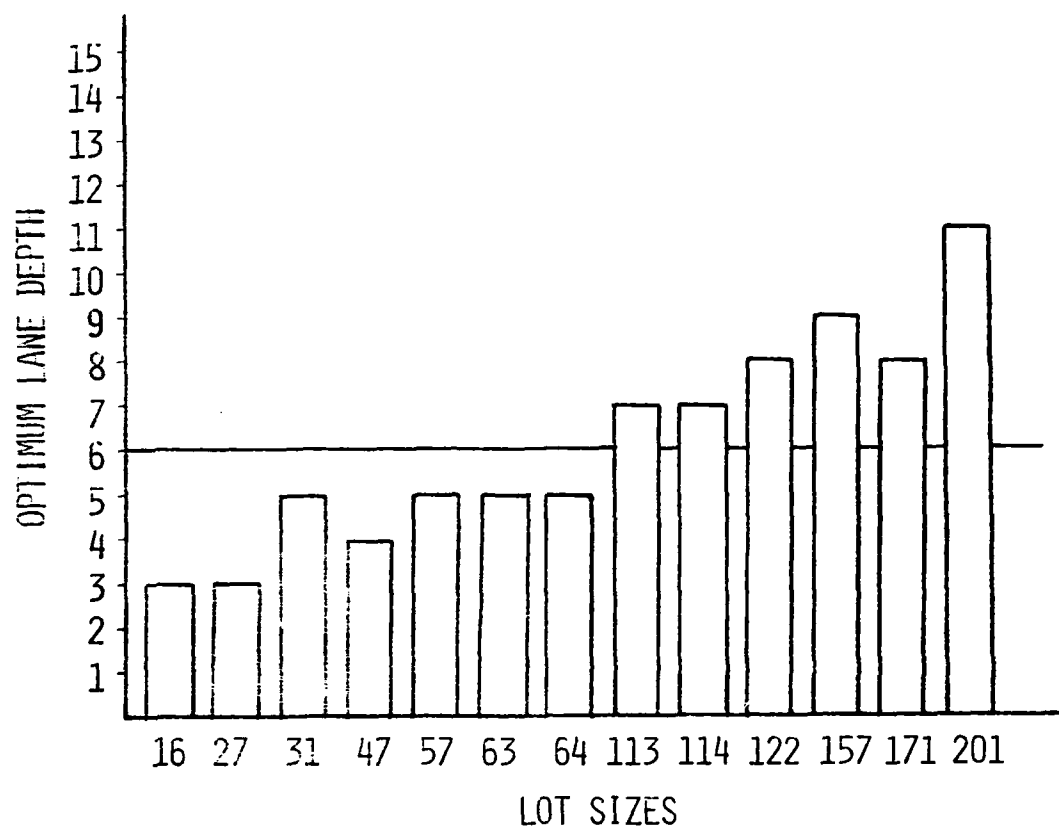


Figure 12. Optimum Lane Depths (Individual and Group)

If the storage-retrieval distributions for all products are characterized by Case 1 assumptions, the optimization problem may be written as

$$\begin{aligned} \text{Minimize} \quad S &= \frac{(W + c)(0.5A + xL)}{288} \sum_{i=1}^n \frac{y_i(2Q_i - xy_iT_i + xT_i)}{Q_i} \\ \text{subject to} \quad xy_iT_i &\geq Q_i \quad \text{for } i = 1, 2, \dots, n \end{aligned}$$

This formulation allows the stacking height to vary for different products. The solution is obtained by total enumeration over feasible values of  $x$ . If lane depth is limited to thirty stacks, a maximum of thirty calculations of the objective function is required to obtain the optimum single lane depth. For the set of thirteen products illustrated in Figure 12 the optimum single lane depth for all products is six pallet stacks, although none of the thirteen products has an individual optimum of six.

Many storage situations will require a design with more than one and less than  $n$  lane depths. Determination of the optimum number and values of lane depths for such a situation is a complex, combinatoric problem. Increasing the number of allowable lane depths from one to two greatly increases the computational requirements. For example, if lane depth is limited to thirty pallet stacks or less, there are 435 possible combinations of two different lane depths. The optimization model for two lane depths is given by:

$$\begin{aligned} \text{Minimize} \quad S &= \frac{(W + c)}{288} \sum_{i=1}^n \left\{ \delta_i \left[ \frac{y_{1i}(0.5A + x_1L)(2Q_i - x_1y_{1i}T_i + x_1T_i)}{Q_i} \right] \right. \\ &\quad \left. + (1 - \delta_i) \left[ \frac{y_{2i}(0.5A + x_2L)(2Q_i - x_2y_{2i}T_i + x_2T_i)}{Q_i} \right] \right\} \end{aligned}$$

subject to  $x_1 y_{1i} T_i \geq Q_i \quad i = 1, 2, \dots, n$

$x_2 y_{2i} T_i \geq Q_i \quad i = 1, 2, \dots, n$

$$\delta_i = \begin{cases} 1 & \text{if product } i \text{ has depth } x_1, \\ 0 & \text{if product } i \text{ has depth } x_2 \end{cases} \quad i = 1, 2, \dots, n$$

#### Problem Extensions and Other Space Requirement Considerations

The space requirements models presented in this report are not a comprehensive group of models for every storage situation. Rather, the report provides a representative group of models for various storage-retrieval distributions and equipment alternatives as well as the methodology used in development of the models. Numerous opportunities exist for further study. For example, a useful and appropriate extension of the work might include an investigation of the optimal lane depths when the product lot can be split over different storage depths. Another study might focus on the development of an economic lot size which uses floor space cost as the criterion.

One aspect of storage system design that is not addressed in this report is the implementation of model results into warehouse layouts. Layout considerations should include:

- building constraints such as columns, doors, and docks;
- location of main aisles and cross-aisles; and
- integration of different handling and storage systems.

Further study is needed for the development of a systematic approach in designing warehouse layouts for storage systems.

#### Handling Requirements

Space utilization is not necessarily the only criterion used by the warehouse planner in the selection and design of storage systems. Handling

times may also be considered in the design decision, since the storage alternative must accommodate throughput requirements. Also, the design that maximizes space utilization can be quite different from the design that minimizes handling times.

In block stacking storage systems, the time required for storage and retrieval of pallet loads may greatly influence the optimum storage lane depth. In developing a model for block stacking handling requirements, the following assumptions are made.

1. A lift truck is used for storage and retrieval of a product lot.
2. The lift truck accommodates only single pallet load moves; thus  $Q$  storage moves and  $Q$  retrieval moves are required for a product lot, regardless of the storage-retrieval distribution.

3. Travel is defined along three dimensions:

$d_x$  = distance moved within the storage lane (ft.)

$d_y$  = distance moved within the aisles (ft.)

$d_z$  = vertical distance moved (ft.)

4. Handling times are a function of distance moved such that the time along each dimension is given by

$$t_x = f(d_x)$$

$$t_y = f(d_y)$$

$$t_z = f(d_z)$$

5. Other elemental times that may be considered include:

$t_t$  = time required to turn from the aisle into the storage lane

$t_p$  = time required to pick up or deposit a pallet load

6. When a product lot is to be stored in  $y$  locations, pallets are stored beginning in the nearest available lane; therefore any partially filled lane will be the farthest of the  $y$  locations.
7. When loads are withdrawn from the lot, loads from any partially filled lane are withdrawn first.

The travel required during the storage-retrieval cycle is illustrated in Figure 13. Each storage requires the lift truck to move the pallet load from a reference point to the storage location, deposit the load, and then return to the reference point. Each retrieval requires the lift truck to travel from the reference point to the retrieval location, pick up the load, and return it to the reference point. Therefore, the lift truck travels the distance between the reference point and any storage location a total of four times during a complete storage-retrieval cycle.

For a given stacking height, the vertical handling time  $t_z$  is a constant since the number of pallet stacks and their vertical configuration are the same for all lane depths. The total time required for vertical handling during a complete storage-retrieval cycle is given by

$$t_z = 4 \left\{ x(y-1) \sum_{i=0}^{T-1} f(iH) + \sum_{i=1}^{Q-x(y-1)T} f\{H[(i-1) \bmod T]\} \right\}$$

where  $f(d_z)$  = a function of distance. The total time required for travel within the storage lane is

$$t_x = 4 \left\{ (y-1)T \sum_{i=0}^{x-1} f(iL) + \sum_{i=1}^{Q-x(y-1)T} f\left[L\left(x - \left\lceil \frac{i}{T} \right\rceil\right)\right] \right\}$$

The distance traveled within the aisles is influenced by the storage configuration, i.e. the number of storage lanes. The total time required for aisle travel during the storage-retrieval cycle which can be attributed to the storage configuration of the given product is

$$t_y = 4 \left\{ xT \sum_{i=0}^{y-1} f[i(W+c)] + [Q - x(y-1)T]f[y(W+c)] \right\}$$

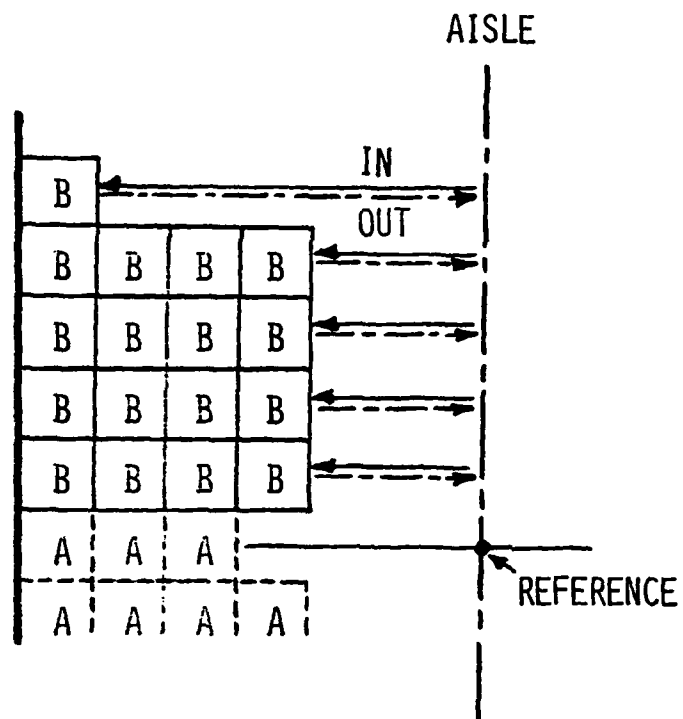


Figure 13. Illustration of Travel Required During a Storage-Retrieval Cycle

The handling times  $t_x$ ,  $t_y$ , and  $t_z$  are functions of distances; the functions are dependent upon such factors as the weight of the load and the equipment used. Other handling times such as  $t_t$  and  $t_p$  are also dependent on the load weight and equipment type but are not dependent on the lane depth. Therefore, for a given stacking height, it is necessary to consider only the variable handling time  $H_v$  (where  $H_v = t_x + t_y$ ) in determining the optimum lane depth with respect to handling. However, total handling times must be analyzed for comparisons of handling requirements for different storage alternatives.

Table 10 provides an example which illustrates the determination of the optimum lane depth for handling times. The example assumes a linear relationship between time and distance for aisle travel; for movement within the storage lane, a power function is assumed. (This power function was determined from preliminary results of an investigation of the influence of lane depth on handling times, by N. A. DeMars of Chesebrough-Pond's, Inc.)

#### Total Cost Considerations

The selection of a storage alternative should be based on an evaluation of space, equipment, and labor costs. An annual cost model for block stacking which includes these costs is given by

$$C_T = C_1 S + C_2 H_v N / 60$$

where

$C_T$  = annual cost for floor space and handling

$C_1$  = floor space cost (\$/ft.<sup>2</sup>-yr.)

$C_2$  = combined hourly cost of lift truck and operator, including overhead (\$/hr.)

Table 10. Block Stacking Example: Determination of Optimum Lane Depth for Handling

Q = 15	L = 50	$t_x = 4 \left\{ (y-1)T \sum_{i=0}^{x-1} .02673(iL)^{.66079} + \sum_{i=1}^{Q-x(y-1)T} .02673 \left[ L \left( x - \left\lceil \frac{i}{T} \right\rceil \right) \right]^{.66079} \right\}$ $t_y = 4 \left\{ xT \sum_{i=0}^{y-1} .00354 [i(W+c)] + [Q - x(y-1)T](.00354)[y(W+c)] \right\}$			
T = 3	W = 42				
	h = 54				
	c = 10				
	A = 144				
x	y	$t_x$	$t_y$	$H_v$	
1	5	0.0000	2.7612	2.7612	OPTIMUM
2	3	2.4709	1.6567	4.1276	
3	2	4.2515	1.2886	5.5401	
4	2	5.5301	1.1045	6.6346	
5	1	5.8865	0.9204	6.8069	

$S$  = average floor space requirements (ft.<sup>2</sup>)

$H_v$  = handling requirements (minutes/storage-retrieval cycle)

$N$  = number of storage-retrieval cycles per year.

Table 11 provides an example which uses both floor space and handling time results from previous examples for Case 1 assumptions (Tables 1 and 10) to determine the optimum lane depth.

Total cost models for other alternatives can be developed in a similar manner. However, the annual equivalent cost of racks or other equipment must also be included in the total cost equation, and any comparison

Table 11. Block Stacking Example: Determination of the Optimum Lane Depth Based on Annual Floor Space and Handling Costs

Table 11. Block Stacking Example: Determination of the Optimum Lane Depth Based on Annual Floor Space and Handling Costs				
Q = 15	L = 50	$S = \frac{y(W + c)(0.5A + xL)(2Q - xyT + xT)}{288Q}$		
T = 3	W = 42			
C <sub>1</sub> = \$2.25	c = 10	$C_T = C_1S + C_2H_v N/60$		
C <sub>2</sub> = \$15.00	A = 144			
N = 20	h = 54			
x	y	S	H <sub>v</sub>	C <sub>T</sub>
1	5	132.17	2.7612	311.18
2	3	111.80	4.1276	272.19 OPTIMUM
3	2	112.23	5.5401	280.23
4	2	117.87	6.6346	298.37
5	1	116.28	6.8069	295.66

between different storage methods must be based on total rather than variable handling times.

#### Analysis of Results

In designing a layout, the warehouse planner may be confronted with building constraints which prohibit implementation of the optimum results of space and handling requirements models. Or, a warehouse manager may need to determine the effect of an increase in handling costs on the optimum lane depth for block stacking. In both of these situations, as well as others, the decision-makers would benefit from a knowledge of the characteristics and sensitivity of the storage system design solutions. This section provides an analysis of numerical results obtained from the

application of some floor space and handling requirements models for block stacking.

### Characteristics of Block Stacking Space Requirements Models

Properties of the Case 1 model. The solution of the single product block stacking space requirements model for Case 1 assumptions is illustrated in Table 12 for a product with  $Q = 147$ . As shown, some characteristics of the space requirements function include the following.

- $S$  is not convex with respect to lane depth  $x$ .
- For a given maximum number of storage lanes,  $y$ , the optimum lane depth  $x$  is not necessarily the smallest feasible depth (such that  $x = \left\lceil \frac{Q}{yT} \right\rceil$  holds). For example, the  $(x, y)$  combination of  $(8, 7)$  requires less floor space on average than the combination  $(7, 7)$ . The combination  $(8, 7)$  is also optimum for all feasible values of  $x$ .
- Several lane depths provide solutions which are "near-optimum." If consideration of near-optimum solutions is arbitrarily limited to those which require an area within ten square feet of the optimum requirements, this example has four near-optimum lane depths ( $x = 7, 10, 9, \text{ or } 6$ ).

Table 13 illustrates optimum and near-optimum solutions for a range of product lot sizes given by DeMars [1980]; Figure 14 provides a graphical representation. These results indicate that, for large lot sizes, the space requirements function is relatively constant for a range of lane depths near the optimum. For example, selection of any of the six near-optimum designs for a product whose lot size is 201 would result in an increase of floor space requirements of at most 1.25% over optimum requirements. This characteristic should be quite helpful in determining lane depths for the case of multiple products. For the products given in Table 13, a lane depth of eight pallet loads could be chosen for products with lot sizes 113, 114, 122, 157, 171, and 201 with little increase in

Table 12. Block Stacking Example: Single Product, Case 1 Assumptions

<p>Q = 147      L = 50</p> <p>T = 3      W = 42</p> <p>c = 10</p> <p>A = 144</p>					
x	y	S (ft. <sup>2</sup> )	x	y	S (ft. <sup>2</sup> )
1	49	1101.4	16	4	642.6
2	25	792.2	17	3	652.3
3	17	695.3	18	3	666.2
4	13	651.5	19	3	677.9
5	10	628.9	20	3	687.3
6	9	616.8	21	3	694.6
7	7	609.6	22	3	699.6
8	7	608.7	23	3	702.4
9	6	611.7	24	3	703.1
10	5	611.2	25	2	711.2
11	5	618.8	26	2	728.0
12	5	619.1	27	2	744.1
13	4	627.9	28	2	759.4
14	4	637.2	29	2	773.9
15	4	642.1	30	2	787.8

floor space requirements.

One operational question of interest might be whether to store in shorter or longer lanes when the optimum lane depth is not available or is not a feasible alternative. However, Table 13 indicates that one cannot state a general rule for this question; the answer is dependent on the particular lot size.

Another observation from Table 13 is that the optimum lane depths

Table 13. Optimum and Near-Optimum Solutions

<p>T = 3</p> <p>L = 50"      c = 10"</p> <p>W = 42"      A = 144"</p>				
Q	S	x	Near-Optimum Depths*	Kind's Approximation
1	44.06	1	None	1
2	44.06	1	None	1
3	44.06	1	None	1
4	55.07	1	2	1
5	61.68	1	2	1
6	62.11	2	1	1
7	70.98	2	1, 3	1
8	77.64	2	3, 1	1
9	80.17	3	2, 1	1
10	86.96	2	3, 1	2
11	90.34	2	3, 4	2
12	93.17	2	4, 3	2
15	111.80	2	3, 5, 4	2
16	115.24	3	2, 4, 5	2
27	160.33	3	4, 5	4
31	180.04	5	4, 3	4
47	242.42	4	5, 6, 3	5
57	281.51	5	6, 4, 7	6
63	304.54	5	7, 6, 4	6
64	308.86	5	7, 6, 4	6
113	489.53	7	8, 6, 9, 10, 5	9
114	493.26	7	8, 6, 9, 10, 5	9
122	519.72	8	7, 6, 10, 9	9
157	644.74	9	8, 10, 7, 11, 6	11
171	693.74	8	9, 10, 11, 7, 12	11
201	797.87	11	10, 9, 8, 12, 13, 7	12

\*in order of nearness to optimum

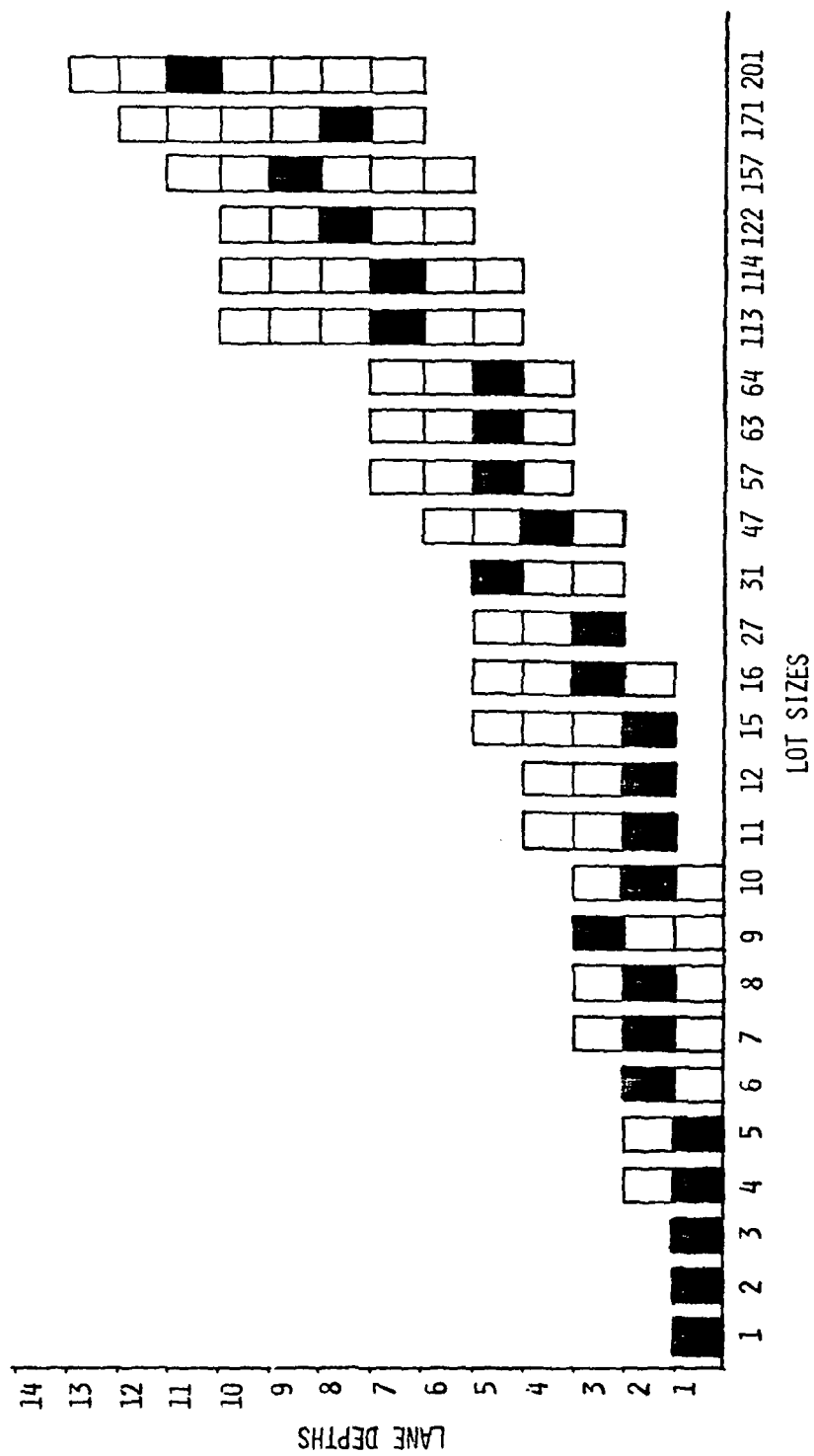


Figure 14. Optimum (Shaded) and Near-Optimum Lane Depths

are relatively small even for larger lot sizes.

Kind's approximation. An approximate formula for determining the optimum lane depth was reported by Kind [1965, 1975]. This approximation was apparently the earliest approach to include the effects of honeycomb loss on space utilization. The lane depths determined by Kind's approximation are included in Table 13.

A comparison of Kind's approximation with the optimum lane depth indicates that his approximation generally over-estimates the lane depth for larger lot sizes. However, the approximation does result in a near-optimum solution.

Effect of withdrawal size. To examine the effect of withdrawal size  $w$  (for a uniform rate of withdrawal), Table 14 provides some results for a product with  $Q = 96$ . The lost sales case assumes that when the lot has less than  $w$  units in stock, an order for  $w$  units is filled with the remaining units in stock. Units from a different lot cannot be used. The backorders case assumes that units in an incoming lot are used to complete any partially filled orders before the new lot is placed in storage. Table 14 illustrates that the optimum lane depth is constant for a wide range of withdrawal sizes, particularly for the backorders case.

For the lost sales case, the number of pallets per lane, using the optimum lane depth, is often close to a multiple of the withdrawal size. For example, when  $w = 16$ , the number of pallets in the optimum storage lane is 33 (since  $xT = 33$ ), or approximately twice the withdrawal size. A rule of thumb sometimes used by warehouse managers for selecting a storage lane depth is to select a lane depth which is a multiple of shipment size. These results tend to validate that rule of thumb for the lost sales case.

Table 14. Effect of Withdrawal Size

$Q = 96$ $L = 50''$ $T = 3$ $W = 42''$ $c = 10''$ $A = 144''$		
Withdrawal Size (w)	Optimum Lane Depth (x) (Lost Sales Case)	Optimum Lane Depth (x) (Backorders Case)
1	8	8
2	8	8
3	8	8
4	8	8
5	8	8
6	8	8
7	7	8
8	8	8
9	8	8
10	8	8
12	8	8
14	7	8
15	9	8
16	11	11
18	8	8
20	7	8
24	8	8
25	8	8
30	12	8
32	11	11
36	11	8
40	11	8
48	16	16

Storage method comparisons. Table 15 provides a comparison of space requirements for four storage alternatives under Case 1 assumptions. The results of Table 15 are based on the assumption that products can be stacked three high for block stacking but four high for racks and deep lane storage alternatives. The minimum space alternative is indicated for each lot size.

A wide range of handling equipment is available for the various storage alternatives. The selection of handling equipment affects both the stacking height and the aisle width. Table 16 provides a comparison of storage methods for various aisle widths and stacking heights which are representative of those for different handling alternatives. [Note that results for AS/RS, one deep and two deep, are given. The AS/RS space requirements are determined from the deep lane storage model by setting  $x = 1$  for one deep AS/RS and  $x = 2$  for two deep AS/RS.]

#### Floor Space vs. Handling Requirements

Table 17 provides an example of floor space and variable handling requirements for block stacking ( $Q = 147$ ). Aisle travel is assumed to be a linear function of distance while travel within the storage lane is represented by a power function. Product turnover is at a rate of twelve storage-retrieval cycles per year.

Effect of turnover rate. For  $C_1 = \$2.25$  and  $C_2 = \$15.00$ , Table 17 illustrates that the optimum lane depth is 7 when there are twelve storage-retrieval cycles per year. The optimum remains 7 pallet loads for turnover rates from 2 to 24 storage-retrieval cycles per year. For a rate of one cycle per year, the optimum lane depth is 8. For 25 to 46 cycles per year, the optimum is 6.

Table 15. Storage Method Space Comparisons

L = 50" W = 42"	Block Stacking		Single-Deep Racks	Double-Deep Racks	Deep Lane Storage System	
	T = 3 c = 10" A = 144"		T = 4 c = 4" f = 6" r = 3"	T = 4 c = 4" f = 6" r = 3"	T = 4 c = 3" f = 6" r = 3"	
Q	x*	S	S	S	x*	S
1	1	44.06	<u>10.74</u>	15.04	1	11.07
2	1	44.06	16.11	<u>15.04</u>	2	15.49
3	1	44.06	21.48	20.05	3	<u>19.92</u>
4	1	55.07	26.86	<u>22.56</u>	2	23.24
5	1	61.68	32.23	<u>27.07</u>	2, 3	27.89
6	2	62.11	37.60	30.08	3	<u>29.88</u>
7	2	70.98	42.97	34.38	3	<u>34.15</u>
8	2	77.64	48.34	37.60	4	<u>36.52</u>
9	3	80.17	53.71	41.78	3	<u>39.84</u>
10	2	86.96	59.08	45.12	5	<u>43.16</u>
11	2	90.34	64.45	49.22	4	<u>46.48</u>
12	2	93.17	69.82	52.64	4	<u>48.70</u>
15	2	111.80	85.94	64.17	5	<u>57.55</u>
16	3	115.24	91.31	67.68	4	<u>60.87</u>
17	3	160.33	150.39	109.17	7	<u>91.98</u>
31	5	180.04	171.88	124.19	6	<u>102.82</u>
47	4	242.42	257.81	184.31	8	<u>144.96</u>
57	5	281.51	311.52	221.89	8	<u>171.18</u>
63	5	304.54	343.75	244.44	9	<u>185.94</u>
64	5	308.86	349.12	248.14	9	<u>188.84</u>
113	7	489.53	612.30	432.41	14	<u>311.52</u>
114	7	493.26	617.68	436.13	13, 14	<u>314.21</u>
122	8	519.72	660.64	466.21	14, 15	<u>334.10</u>
157	9	644.74	848.63	597.83	16	<u>419.44</u>
171	8	693.74	923.83	650.46	17	<u>453.09</u>
201	11	797.87	1084.96	763.25	17	<u>525.63</u>

Table 16. Comparison of Alternative Storage Methods for Case 1 Assumptions

Storage Method	Aisle Width (ft.) # Levels (z)	Block Stacking		Deep Lane				Single Deep Pallet Rack				Double Deep Pallet Rack				AS/RS 1-Deep				AS/RS 2-Deep			
		12 3		6 6		6 10		8 5		6 6		6 8		8 5		6 10		6 12		6 10		6 12	
		x	s	x	s	x	s	x	s	x	s	x	s	x	s	x	s	x	s	x	s	x	s
5		1,2	56.00	2	14.38	2	8.63	19.76	14.53	10.90	17.44	13.95	10.46	9.37	7.81	8.63	7.19						
10		2	78.40	3	23.68	3	14.21	36.23	26.64	19.98	29.06	23.25	17.44	17.19	14.32	14.38	11.98						
15		2,3	100.80	3	32.29	3	19.88	52.70	38.75	29.06	41.33	33.07	24.80	25.00	20.83	20.44	17.04						
20		3	118.80	4	40.63	4	24.38	69.17	50.86	38.14	53.28	42.63	31.97	32.81	27.34	26.35	21.96						
25		4	137.28	5	48.96	5	29.38	85.64	62.97	47.23	65.49	52.39	39.29	40.63	33.85	32.39	26.99						
30		5	156.00	5	57.12	5	34.27	102.11	75.08	56.31	77.50	62.00	46.50	48.44	40.36	38.33	31.94						
35		4	173.49	5	65.28	5	39.17	118.58	87.19	65.39	89.68	71.74	53.81	56.25	46.88	44.36	36.96						
40		4	193.60	5	73.44	5	44.06	135.04	99.30	74.47	101.72	81.37	61.03	64.06	53.39	50.31	41.93						
45		5	208.00	6	81.48	6	48.89	151.51	111.41	83.55	113.88	91.11	68.33	71.88	59.90	56.33	46.94						
50		4,5	228.80	7	89.25	7	53.55	167.98	123.52	92.64	125.94	100.75	75.56	79.69	66.41	62.29	51.91						
60		5	260.00	7	105.00	7	63.00	200.92	147.73	110.80	150.16	120.13	90.09	95.31	79.43	74.27	61.89						
70		6	294.86	7	120.31	7	72.19	231.86	171.95	128.96	174.38	139.50	104.63	110.94	92.45	86.25	71.88						
80		7	329.80	8	135.59	8	81.35	266.79	196.17	147.13	198.59	158.88	119.16	126.56	105.47	98.23	81.86						
100		7	394.40	10	166.15	10	99.69	332.67	244.61	183.46	247.03	197.63	148.22	157.81	131.51	122.19	101.82						
125		7	473.28	10	204.21	10	122.53	415.01	305.16	228.87	307.60	246.08	184.56	196.88	164.06	152.15	126.79						
150		10	552.00	10	241.67	10	145.00	497.36	365.70	274.28	368.13	294.50	220.88	235.94	196.61	182.08	151.74						
175		10	630.86	12	278.96	11,12	167.38	579.70	426.25	319.69	428.69	342.95	257.21	275.00	229.17	212.04	176.70						
200		11	707.00	13	316.04	13	189.63	662.04	486.80	365.10	489.22	391.38	293.53	314.06	261.72	241.98	201.65						
225		11	784.00	14,15	352.78	14,15	211.67	744.39	547.34	410.51	549.78	439.82	329.87	353.13	294.27	271.93	226.61						
250		12	858.82	14	389.73	14	233.84	826.73	607.89	455.92	610.31	488.25	366.19	392.19	326.82	301.88	251.56						
300		11	1010.00	15	463.02	15	277.81	991.42	728.98	546.74	731.41	585.12	438.84	470.31	391.93	361.77	301.48						
400		15	1306.80	19	608.81	19	365.29	1370.79	971.17	728.38	973.59	778.87	584.16	626.56	522.14	481.56	401.30						
500		15	1599.84	20	753.82	20	452.29	1650.17	1213.36	910.02	1215.78	972.62	729.47	782.61	652.34	601.35	501.13						

Data: L = 48 in., W = 40 in., r = 4 in., c = 8 in., c' = 3 in., c'' = 3 in., f = 12 in.

Table 17. Block Stacking Example. Floor  
Space vs. Handling Costs

$$\begin{aligned}
 Q &= 147 & L &= 50 \\
 T &= 3 & W &= 42 \\
 N &= 12 & c &= 10 \\
 C_1 &= \$2.25 & A &= 144 \\
 C_2 &= \$15.00
 \end{aligned}$$

$$t_x = 4 \left\{ (y-1)T \sum_{i=0}^{x-1} .02673(iL) \cdot .66079 \right.$$

$$\left. + \sum_{i=1}^{Q-x(y-1)T} .02673 \left[ L \left( x - \left\lceil \frac{i}{T} \right\rceil \right) \right] \cdot .66079 \right\}$$

$$t_y = 4 \left\{ xT \sum_{i=0}^{y-1} .00354[i(W+c)] \right.$$

$$\left. + [Q - x(y-1)T] \cdot .00354[y(W+c)] \right\}$$

x	y	S	H <sub>v</sub>	C <sub>T</sub>
1	49	1101.39	225.49800	3154.62
2	25	792.23	135.64067	2189.45
3	17	695.32	113.54792	1905.12
4	13	651.47	107.46329	1788.21
5	10	628.85	107.64615	1737.85
6	9	616.84	109.98062	1717.82
7	7	609.56	112.82203	1709.97
8	7	608.73	118.85059	1726.19
9	6	611.66	126.56614	1755.94
10	5	611.24	130.26773	1766.08
11	5	618.83	139.01258	1809.40
12	5	619.05	141.04075	1815.98
13	4	627.86	150.62750	1864.57
14	4	637.21	158.62765	1909.60
15	4	642.13	162.84217	1933.32
16	4	642.63	163.99184	1937.89
17	3	652.30	172.93707	1986.48
18	3	666.18	183.38842	2049.08
19	3	677.86	191.46473	2099.57
20	3	687.32	197.66030	2139.45

Sensitivity to cost estimates. For  $N = 12$ , the optimum lane depth is affected by changes in floor space or handling costs as follows:

- For  $C_2 = \$15.00$ , the optimum lane depth is 7 pallet loads for  $1.18 \leq C_1 \leq 21.79$
- For  $C_1 = \$2.25$ , the optimum lane depth is 7 pallet loads for  $1.54 \leq C_2 \leq 28.82$

Therefore, the optimum is relatively insensitive to minor changes in either floor space or handling costs.

#### Conclusion

This report has illustrated the development and application of analytical models, based on floor space and handling time criteria, for some selected storage alternatives. Results indicate that significant savings in storage costs may be achieved through the appropriate use of such models in the storage design process.

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Appendix

### Derivation of Block Stacking Space Requirements Models

A block stacking storage lane for any storage-retrieval distribution has a depth of  $(0.5A + xL)$  inches, a width of  $(W + c)$  inches, and a height of  $T$  pallet loads. The average floor space required for block stacking is thus given by

$$S_k = \frac{(W + c)(0.5A + xL)}{144} \eta_k$$

where

$\eta_k$  = average number of storage lanes required for Case  $k$ ,

and  $S_k$  = average floor space requirements for Case  $k$ .

The derivation of  $\eta_k$  for each of the three storage-retrieval assumptions is illustrated in the following paragraphs.

Case 1. Table A1 summarizes the lane requirements and corresponding time requirements for Case 1 assumptions.

Table A1. Storage Lane Requirements for Block Stacking, Case 1	
Number of Lanes	Time Required
$y$	$Q - xT(y - 1)$
$y - 1$	$xT$
$y - 2$	$xT$
$\vdots$	$\vdots$
$y - j$	$xT$
$\vdots$	$\vdots$
2	$xT$
1	$xT$
Total Time	$Q$

The average number of lanes determined from Table A1 is

$$\eta_1 = \frac{y[Q - xT(y-1)]}{Q} + \sum_{j=1}^{y-1} (y-j) \frac{xT}{Q}$$

Simplifying and combining terms,

$$\eta_1 = \frac{y(2Q - xyT + xT)}{2Q}$$

Therefore, the average block stacking floor space requirements for Case 1 are given by

$$S_1 = \frac{y(W + c)(0.5A + xL)(2Q - xyT + xT)}{288Q}$$

Case 2. The lane and time requirements for block stacking under Case 2 assumptions are given in Table A2.

From Table A2, the average number of lanes for Case 2 is determined by

$$\eta_2 = \frac{y \left[ \frac{1 - p^{Q-xT(y-1)}}{1 - p} \right] + \sum_{j=1}^{y-1} (y-j) \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-xT(y-j)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

which reduces to

$$\eta_2 = \left( \frac{1}{1 - p^Q} \right) \left[ y - p^{Q-xT(y-1)} \left( \frac{1 - p^{xTy}}{1 - p^{xT}} \right) \right]$$

The average block stacking floor space requirements for Case 2 are

$$S_2 = \frac{(W + c)(0.5A + xL)}{144(1 - p^Q)} \left[ y - p^{Q-xT(y-1)} \left( \frac{1 - p^{xTy}}{1 - p^{xT}} \right) \right]$$

Number of Lanes	Time Required
y	$\sum_{k=0}^{Q-xT(y-1)-1} p^k = \frac{1 - p^{Q-xT(y-1)+1}}{1 - p}$
y - 1	$\sum_{k=Q-xT(y-1)}^{Q-xT(y-2)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-xT(y-1)}$
y - 2	$\sum_{k=Q-xT(y-2)}^{Q-xT(y-3)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-xT(y-2)}$
$\vdots$	$\vdots$
y - j	$\sum_{k=Q-xT(y-j)}^{Q-xT(y-j-1)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-xT(y-j)}$
$\vdots$	$\vdots$
2	$\sum_{k=Q-2xT}^{Q-xT-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-2xT}$
1	$\sum_{k=Q-xT}^{Q-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{Q-xT}$
Total Time	$\left( \frac{1 - p^Q}{1 - p} \right)$

Case 3. Table A3 provides the lane and time requirements for block stacking, given Case 3 assumptions. The average number of lanes required is determined by

$$\eta_3 = \frac{y p^{xT(y-1)} \left[ \frac{1 - p^{Q-xT(y-1)+1}}{1 - p} \right] + \sum_{j=1}^{y-1} (y - j) \left( \frac{1 - p^{xT}}{1 - p} \right) p^{xT(y-j-1)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

or

$$\eta_3 = \left( \frac{1}{1 - p^Q} \right) \left[ \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) - yp^Q \right]$$

Therefore, the average floor space for Case 3 is given by

$$S = \frac{(W + c)(0.5A + xL)}{144(1 - p^Q)} \left[ \left( \frac{1 - p^{xyT}}{1 - p^{xT}} \right) - yp^Q \right]$$

Table A3. Storage Lane Requirements for Block Stacking, Case 3	
Number of Lanes	Time Required
y	$\sum_{k=xT(y-1)}^{Q-1} p^k = \left[ \frac{1 - p^{Q-xT(y-1)}}{1 - p} \right] p^{xT(y-1)}$
y - 1	$\sum_{k=xT(y-2)}^{xT(y-1)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{xT(y-2)}$
y - 2	$\sum_{k=xT(y-3)}^{xT(y-2)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{xT(y-3)}$
⋮	⋮
y - j	$\sum_{k=xT(y-j-1)}^{xT(y-j)-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{xT(y-j-1)}$
⋮	⋮
2	$\sum_{k=xT}^{2xT-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right) p^{xT}$
1	$\sum_{k=0}^{xT-1} p^k = \left( \frac{1 - p^{xT}}{1 - p} \right)$
Total Time	$\left( \frac{1 - p^Q}{1 - p} \right)$

### Derivation of Space Requirements Models for Single-Deep Racks

For single-deep rack storage, a storage slot has a depth of  $[0.5(A + f) + L]$  inches (since  $x = 1$ ), a width of  $(W + 0.5r + 1.5c)$  inches, and a height of one pallet load. There are  $T$  storage levels. The average floor space required for single-deep racks is

$$S_k = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L]}{144T} \eta_k$$

where  $\eta_k$  = average number of storage slots required for Case  $k$ .

Table A4 illustrates the number of storage slots and time requirements for single-deep racks for each of the three storage-retrieval distributions.

Table A4. Storage Slot Requirements for Single-Deep Racks			
Number of Storage Slots	Time Required		
	Case 1	Case 2	Case 3
$v = Q$	1	$p^0$	$p^{Q-1}$
$v - 1$	1	$p^1$	$p^{Q-2}$
$v - 2$	1	$p^2$	$p^{Q-3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v - j$	1	$p^j$	$p^{Q-j-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	1	$p^{Q-2}$	$p^1$
1	1	$p^{Q-1}$	$p^0$
Total Time	$Q$	$\left( \frac{1 - p^Q}{1 - p} \right)$	$\left( \frac{1 - p^Q}{1 - p} \right)$

For Case 1, the average number of storage slots is determined by

$$\eta_1 = \sum_{j=0}^v (v - j) \frac{1}{Q}$$

which reduces to

$$\eta_1 = \frac{Q + 1}{2}, \quad \text{since } v = Q.$$

Therefore, for single-deep racks,

$$S_1 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L](Q + 1)}{288T}$$

For Case 2,

$$\eta_2 = \frac{\sum_{j=0}^{v-1} (v - j)p^j}{\left(\frac{1 - p^Q}{1 - p}\right)}$$

or

$$\eta_2 = \left(\frac{1}{1 - p^Q}\right) \left[Q - \frac{p(1 - p^Q)}{1 - p}\right]$$

Thus, average floor space requirements for single-deep racks under Case 2 assumptions are given by

$$S_2 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L]}{144T(1 - p^Q)} \left[Q - \frac{p(1 - p^Q)}{1 - p}\right]$$

For Case 3,

$$\eta_3 = \frac{\sum_{j=0}^{v-1} (v - j)p^{(v-j-1)}}{\left(\frac{1 - p^Q}{1 - p}\right)}$$

or

$$\eta_3 = \left( \frac{1}{1 - p^Q} \right) \left[ \left( \frac{1 - p^Q}{1 - p} \right) - Qp^Q \right]$$

For single-deep racks,

$$S_3 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + L]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^Q}{1 - p} \right) - Qp^Q \right]$$

#### Derivation of Space Requirements Models for Double-Deep Racks

A storage slot for double-deep racks has a depth of  $[0.5(A + f) + 2L]$  inches (since  $x = 2$ ), a width of  $(W + 0.5r + 1.5c)$  inches, and a height of one pallet load. A product lot may be stored in  $T$  storage levels. Thus the average floor space required for double-deep racks is

$$S_k = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + 2L]}{144T} \eta_k$$

where  $\eta_k$  = average number of storage slots required for Case  $k$ .

Table A5 illustrates the storage slot requirements for the three storage-retrieval assumptions.

For Case 1,

$$\eta_1 = \frac{v[Q - 2(v - 1)]}{Q} + \sum_{j=1}^{v-1} (v - j) \left( \frac{2}{Q} \right)$$

or

$$\eta_1 = \frac{v(Q - v + 1)}{Q}$$

The average floor space requirements for double-deep racks under Case 1 assumptions are given by

$$S_1 = \frac{v(W + 0.5r + 1.5c)[0.5(A + f) + 2L](Q - v + 1)}{144TQ}$$

Table A5. Storage Slot Requirements for Double-Deep Racks

Number of Storage Slots	Time Required		
	Case 1	Case 2	Case 3
$v$	$Q - 2(v - 1)$	$\left[ \frac{1 - p^{Q-2(v-1)}}{1 - p} \right]$	$\left[ \frac{1 - p^{Q-2(v-1)}}{1 - p} \right] p^{2(v-1)}$
$v - 1$	2	$\left( \frac{1 - p^2}{1 - p} \right) p^{Q-2(v-1)}$	$\left( \frac{1 - p^2}{1 - p} \right) p^{2(v-2)}$
$v - 2$	2	$\left( \frac{1 - p^2}{1 - p} \right) p^{Q-2(v-2)}$	$\left( \frac{1 - p^2}{1 - p} \right) p^{2(v-3)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v - j$	2	$\left( \frac{1 - p^2}{1 - p} \right) p^{Q-2(v-j)}$	$\left( \frac{1 - p^2}{1 - p} \right) p^{2(v-j-1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
2	2	$\left( \frac{1 - p^2}{1 - p} \right) p^{Q-4}$	$\left( \frac{1 - p^2}{1 - p} \right) p^2$
1	2	$\left( \frac{1 - p^2}{1 - p} \right) p^{Q-2}$	$\left( \frac{1 - p^2}{1 - p} \right)$
Total Time	$Q$	$\left( \frac{1 - p^Q}{1 - p} \right)$	$\left( \frac{1 - p^Q}{1 - p} \right)$

For Case 2,

$$\eta_2 = \frac{v \left[ \frac{1 - p^{Q-2(v-1)}}{1 - p} \right] + \sum_{j=1}^{v-1} (v - j) \left( \frac{1 - p^2}{1 - p} \right) p^{Q-2(v-j)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

which reduces to

$$\eta_2 = \left( \frac{1}{1 - p^Q} \right) \left[ v - p^{Q-2(v-1)} \left( \frac{1 - p^{2v}}{1 - p^2} \right) \right]$$

For double-deep racks,

$$S_2 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + 2L]}{144T(1 - p^Q)} \left[ v - p^{Q-2(v-1)} \left( \frac{1 - p^{2v}}{1 - p^2} \right) \right]$$

For Case 3,

$$\eta_3 = \frac{vp^{2(v-1)} \left[ \frac{1 - p^{Q-2(v-1)}}{1 - p} \right] + \sum_{j=1}^{v-1} (v - j) \left( \frac{1 - p^2}{1 - p} \right) p^{2(v-j-1)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

or

$$\eta_3 = \left( \frac{1}{1 - p^Q} \right) \left[ \left( \frac{1 - p^{2v}}{1 - p^2} \right) - vp^Q \right]$$

For Case 3, the average floor space requirements for double-deep racks are given by

$$S_3 = \frac{(W + 0.5r + 1.5c)[0.5(A + f) + 2L]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^{2v}}{1 - p^2} \right) - vp^Q \right]$$

For double-deep racks,

$$v = \begin{cases} \frac{Q}{2} & , \quad Q \text{ even} \\ \frac{Q+1}{2} & , \quad Q \text{ odd} \end{cases}$$

or

$$v = \frac{2Q + 1 - (-1)^Q}{4} \quad \text{for any } Q.$$

### Derivation of Space Requirements Models for Deep Lane Storage

In a deep lane storage system, there are  $T$  levels of storage. Each storage slot has a depth of  $[0.5(A + f) + xL]$  inches, a width of  $(W + r + 2c)$  inches, and a height of one pallet load. The average floor space required for deep lane storage is

$$S_k = \frac{(W + r + 2c)[0.5(A + f) + xL]}{144T} \eta_k$$

where  $\eta_k$  = average number of storage slots required for Case  $k$ .

The storage slot requirements for each of the three storage-retrieval assumptions are given in Table A6.

For Case 1,

$$\eta_1 = \frac{v[Q - x(v - 1)]}{Q} + \sum_{j=1}^{v-1} (v - j) \left( \frac{x}{Q} \right)$$

or

$$\eta_1 = \frac{v(2Q - xv + x)}{2Q}$$

Therefore, the average floor space requirements for deep lane storage under Case 1 assumptions are given by

$$S_1 = \frac{v(W + r + 2c)[0.5(A + f) + xL](2Q - xv + x)}{288TQ}$$

For Case 2,

$$\eta_2 = \frac{v \left[ \frac{1 - p^{Q-x(v-1)}}{1 - p} \right] + \sum_{j=1}^{v-1} (v - j) \left( \frac{1 - p^x}{1 - p} \right) p^{Q-x(v-j)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

Table A6. Storage Slot Requirements for Deep Lane Storage

Number of Storage Slots	Time Required		
	Case 1	Case 2	Case 3
$v$	$Q - x(v - 1)$	$\left[ \frac{1 - p^{Q-x(v-1)}}{1 - p} \right]$	$\left[ \frac{1 - p^{Q-x(v-1)}}{1 - p} \right] p^{x(v-1)}$
$v - 1$	$x$	$\left( \frac{1 - p^x}{1 - p} \right) p^{Q-x(v-1)}$	$\left( \frac{1 - p^x}{1 - p} \right) p^{x(v-2)}$
$v - 2$	$x$	$\left( \frac{1 - p^x}{1 - p} \right) p^{Q-x(v-2)}$	$\left( \frac{1 - p^x}{1 - p} \right) p^{x(v-3)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$v - j$	$x$	$\left( \frac{1 - p^x}{1 - p} \right) p^{Q-x(v-j)}$	$\left( \frac{1 - p^x}{1 - p} \right) p^{x(v-j-1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$2$	$x$	$\left( \frac{1 - p^x}{1 - p} \right) p^{Q-2x}$	$\left( \frac{1 - p^x}{1 - p} \right) p^x$
$1$	$x$	$\left( \frac{1 - p^x}{1 - p} \right) p^{Q-x}$	$\left( \frac{1 - p^x}{1 - p} \right)$
Total Time	$Q$	$\left( \frac{1 - p^Q}{1 - p} \right)$	$\left( \frac{1 - p^Q}{1 - p} \right)$

which reduces to

$$\eta_2 = \left( \frac{1}{1 - p^Q} \right) \left[ v - p^{Q-x(v-1)} \left( \frac{1 - p^{xv}}{1 - p^x} \right) \right]$$

Thus, for deep lane storage

$$S_2 = \frac{(W + r + 2c)[0.5(A + f) + xL]}{144T(1 - p^Q)} \left[ v - p^{Q-x(v-1)} \left( \frac{1 - p^{xv}}{1 - p^x} \right) \right]$$

For Case 3,

$$\eta_3 = \frac{vp^{x(v-1)} \left[ \frac{1 - p^{Q-x(v-1)}}{1 - p} \right] + \sum_{j=1}^{v-1} (v - j) \left( \frac{1 - p^x}{1 - p} \right) p^{x(v-j-1)}}{\left( \frac{1 - p^Q}{1 - p} \right)}$$

or

$$\eta_3 = \left( \frac{1}{1 - p^Q} \right) \left[ \left( \frac{1 - p^{xv}}{1 - p^x} \right) - vp^Q \right]$$

The average floor space requirements for deep lane storage under Case 3 assumptions are given by

$$S_3 = \frac{(W + r + 2c)[0.5(A + f) + xL]}{144T(1 - p^Q)} \left[ \left( \frac{1 - p^{xv}}{1 - p^x} \right) - vp^Q \right]$$

